Force and Equilibrium

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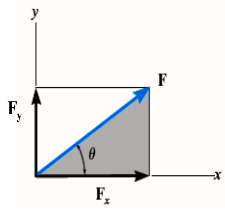
Outline

- Force
- Newton's laws of motion
- Equilibrium

Force

- Force is a vector quantity that causes the change in motion of an object
- The unit is newton (N)
- Examples, weight, push, pull, tension, friction, electromagnetic
- Resolution of force – force can be resolved in two perpendicular components



Common forces

- Weight (W)
- Tension (T)
- Normal contact force (N)
- Friction (f)

Weight

Newton (Newton's Law of Gravitational Attraction)

$$F = G\frac{m_1 m_2}{r^2}$$

F = gravitational force between 2 particles
G = gravitational constant 6.67384 × 10^{-11} m^3 kg^{-1} s^{-2}
m_1, m_2 = masses of particles
r = distance between particles

น้ำหนัก: $W = G\frac{mM_e}{r^2}$

ให้ $g = GM_e / r^2$ จะได้ $W = mg$

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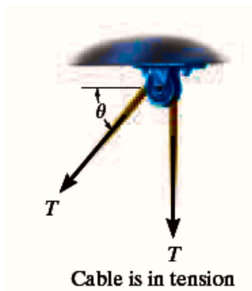
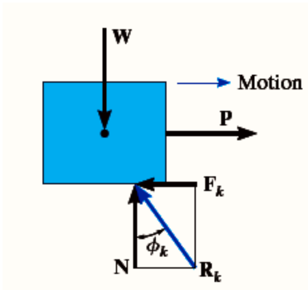
Tension

- This is a force in a stretched string acting on an object by a string
- No self-weight
- The direction of the force in the direction of a cable
- Constant



Normal Contact force

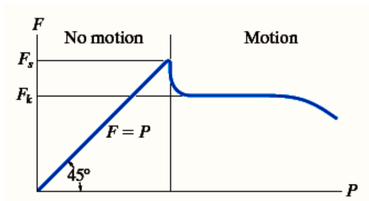
- This is a force between two contact surfaces, exerted on the object by the surface in the direction perpendicular to the surface



Friction

- This is a force between two contact surfaces that slides against each other. Friction is exerted on the object by the surface in the direction opposing the motion. The size of friction is determined by the coefficient of friction µ between the two contact surfaces. The formula is given by f = µN
- There are two types of frictions: static friction and kinetic friction
- Kinetic friction is less than static friction about 25%

$$\phi_k = \tan^{-1}\left(\frac{F_k}{N}\right) = \tan^{-1}\left(\frac{\mu_k N}{N}\right) = \tan^{-1}\mu_k$$

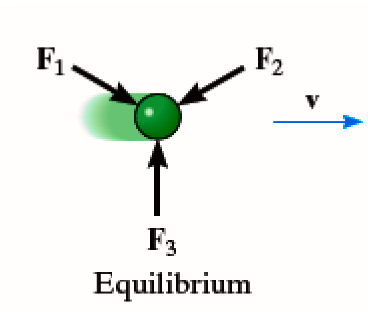


Momentum

- Consider an object with mass m moving with a velocity **v**. The momentum p of the object is defined by **p** = m**v**

Newton's laws of motion

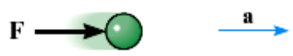
- First law : If the resultant force acting on a body is zero, the body remains either at rest or moving at constant velocity. This law is often known as the "law of inertia"



Newton's laws of motion

- Second law : Force acting on a boy is equal to the rate of change of its momentum
- From the statement, it can be written as $\mathbf{F} = \frac{d\mathbf{p}}{dt}.$
- Substitu $\mathbf{p} = m\mathbf{v}$ into the above equation

$$\mathbf{F} = m\frac{d\mathbf{v}}{dt} + \mathbf{v}\frac{dm}{dt}.$$



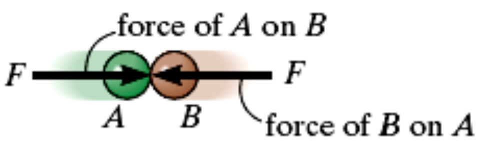
Accelerated motion

- If the mass is constant, then

$$\mathbf{F} = m\frac{d\mathbf{v}}{dt} = m\boldsymbol{a}.$$

Newton's laws of motion

- Third law : If two bodies interact, the force $\mathbf{F}_{12}$ exerted by body 1 on body 2 is equal in magnitude and opposite direction to the force $\mathbf{F}_{21}$ by object 2 on object 1. The third law is often known as action = reaction



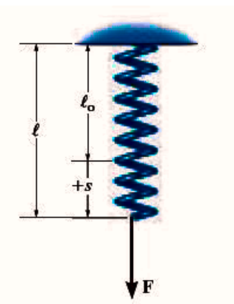
Action – reaction

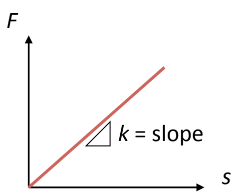
Equilibrium

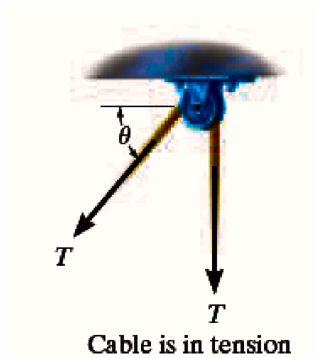
- Particle is equilibrium if it is
  - At rest
  - Moving at a constant velocity
- Newton's first law of motion $\sum \mathbf{F} = 0$
- From Newton's second law of motion $\sum \mathbf{F} = m\mathbf{a}$, $m\mathbf{a} = 0$, hence $\mathbf{a} = 0$
- Therefore the particle is either at rest or moving at a constant velocity

Equilibrium

- To consider an object in equilibrium, a Free body diagram (FBD) is used
- FBD display all forces acting on an objec
- E.g. Spring, Cables and Pulleys

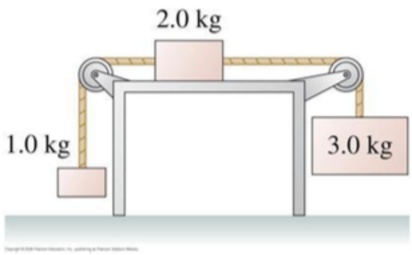






Example

- Consider a setup in the figure below. The table surface is smooth and the strings are inextensible. Determine
  (a) The acceleration of the system
  (b) The tensions in the strings



Example

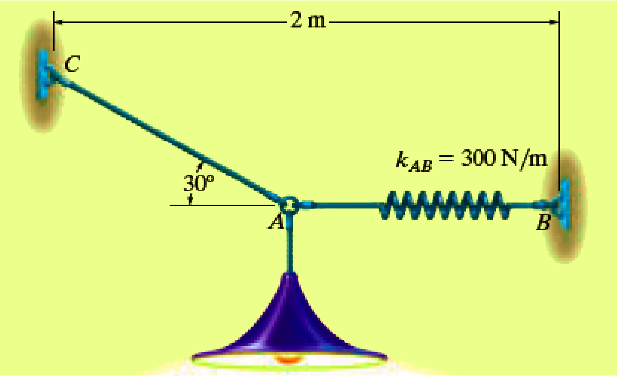
- A body with a mass of 2 kg is exerted by a force $F(t) = 4t^2 - 2t$ where F is measured in N and t in s. If the body starts from velocity 1.5 m/s, the the velocity at time t = 2 s.

Example

- A submarine of mass m moving horizontally under water experiences a drag force F = -kv, where k is a positive contant and v is speed. If the initial speed of the submarine is u, find the function of speed at any other time t

Example

Determine the required length of the cord AC so that the 8kg lamp is suspended. The undeformed length of the spring AB is L'_{AB} = 0.4m, and the spring has a stiffness of k_{AB} = 300N/m.



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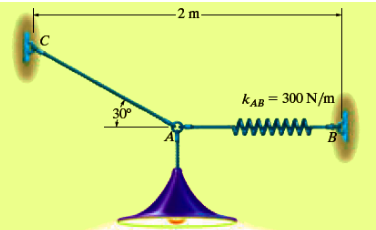
Solution



FBD at Point A

Three forces acting, force by cable AC, force in spring AB and weight of the lamp.

If force on cable AB is known, stretch of the spring is found by $F = ks$.

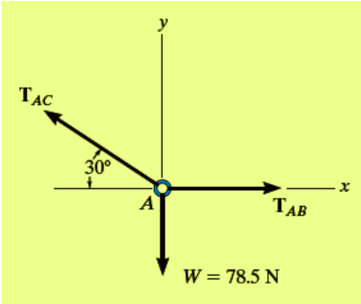
+→ $\sum \mathbf{F}_x = 0$; $T_{AB} - T_{AC}\cos 30^\circ = 0$

+↑ $\sum \mathbf{F}_y = 0$; $T_{AC}\sin 30^\circ - 78.5N = 0$

Solving,

T_{AC} = 157.0kN

T_{AB} = 136.0kN



Solution

$T_{AB} = k_{AB}s_{AB}$; $136.0N$ = 300N/m(s_{AB}) s_{AB} = 0.453 m

For stretched length,

$L_{AB} = L'_{AB} + s_{AB}$

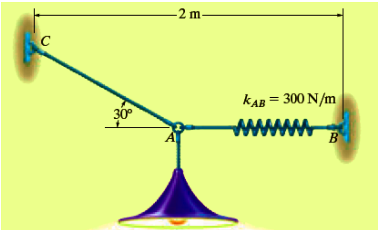
L_{AB} = 0.4m + 0.453m

= 0.853m

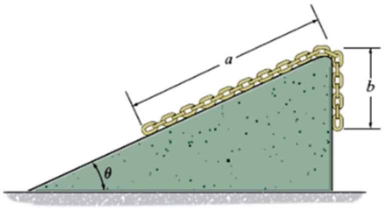
For horizontal distance BC,

2m = $L_{AC}\cos 30^\circ$ + 0.853m

L_{AC} = 1.32m



•8–37. If the coefficient of static friction between the chain and the inclined plane is $\mu_s = \tan\theta$, determine the overhang length b so that the chain is on the verge of slipping up the plane. The chain weighs w per unit length.



Free - Body Diagram. The tension developed in the chain at the end of the inclined plane is equal to the weight of the overhanging chain, i.e. $T = wb$. Since the chain is required to be on the verge of sliding up the plane, the frictional force **F** must act down the plane so that $F = \mu_s N = \tan\theta N$ as indicated on the free - body diagram of the chain shown in Fig. *a*.

Equations of Equilibrium.

$\nwarrow^{+}\Sigma F_{y'} = 0;\quad N - wa\cos\theta = 0 \qquad N = wa\cos\theta$

$+\nearrow\Sigma F_{x'} = 0;\quad wb - wa\sin\theta - \tan\theta(wa\cos\theta) = 0$

$b = 2a\sin\theta$ **Ans.**