Part IB Computing Course — Session 3B

Summary

After reading through Sections 11 and 12 of the tutorial guide and working through the examples you should be able to:

- ☐ Write linear simultaneous equations in matrix-vector form
- ☐ Use the slash operator in MATLAB to solve $\mathtt{A}\mathbf{x} = \mathbf{b}$
- ☐ Understand what the slash operator does in different scenarios
- ☐ Appreciate the problems of ill-conditioned matrices
- ☐ Produce more advanced figures, including displaying three-dimensional data

Exercise 6: Leg Before Wicket?

A. Description

The Leg Before Wicket (LBW) rule in the game of cricket is the cause of many contentious decisions, but the problem is a very simple one: if the batsman had not got in the way, would the ball have hit the stumps? This is an extrapolation problem. We can track the ball's position from the point it bounces, until it hits the batsman, and the problem is then just one of fitting a curve to the observed trajectory and extrapolating this to see if it hits the stumps.

A tracking system measures the ball's position at a rate of 150Hz, and for each sample stores the x, y and z co-ordinate of the ball. Measurements are in metres, according to the axes defined in Figure 12, centred on the bottom of the middle stump. The ball's position is accurate to about 2cm.

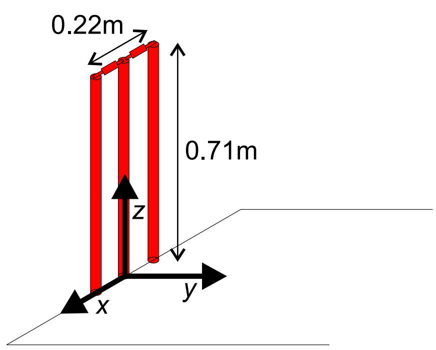

Figure 12: Definition of axes, and dimensions of the stumps

B. Exercise

The file `cricket.txt` contains data from a disputed decision, to be analysed by this system. The file contains 4 columns of data, in the format `(`*`time x y z`*`)`. Load this file into MATLAB and produce a three dimensional plot of the position data, marking each sample with a blob.

Assuming a linear horizontal trajectory (i.e. that the ball's velocity is constant in x and y), and that its height describes a parabola, express each of the three co-ordinates as a function of time. Use least squares to estimate the motion parameters and so fit a curve to the sample points. *You should not assume you know the value of the acceleration due to gravity, g, and you should instead include this as one of your unknown parameters.* What value for g is given by this model? Add your estimated curve to the 3D plot, extrapolating it past the stumps.

Fit another curve by least squares, this time assuming that $g = 9.81$, and add this to your plot in a different colour. Do you notice a significant difference? Using this model of the ball's trajectory, what was the velocity of the ball when $t = 0$? Finally, calculate the ball's position as it passes the stumps. Would the ball have hit the stumps?

C. Implementation notes

1. **Solving equations of motion by least squares**
   For each sample point you can write three equations in terms of time: one for x, one for y, and one for z. The first two are linear, in the form

   $$x = x_0 + u_x t$$

   And similarly for y. Each of these equations has two parameters, the speeds u_x and u_y, and the initial positions x_0 and y_0. The z motion will have an additional quadratic term involving g:

   $$z = z_0 + u_z t + \frac{1}{2} g t^2$$

   If g is unknown (as in the first part of this exercise), then this has three unknown parameters, giving seven parameters in total. The three equations may be written in matrix-vector form as

   $$\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \left[\quad \mathtt{A} \quad \right] \begin{pmatrix} u_x \\ u_y \\ u_z \\ x_0 \\ y_0 \\ z_0 \\ g \end{pmatrix}$$

   where the elements in `A` are given by a straightforward comparison of coefficients with the three motion equations.

   Each sample point gives different values of x, y and z, and so three different equations. Each of these equations gives different values for the matrix `A`, but the parameters u_x etc. *stay exactly the same* (these are our initial conditions, and define the fitted trajectory). We can therefore stack all of our equations on top of each other to create some huge matrices and vectors:

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \\ \hline x_2 \\ y_2 \\ z_2 \\ \hline \vdots \\ \hline x_n \\ y_n \\ z_n \end{pmatrix} = \begin{bmatrix} \mathtt{A_1} \\ \hline \mathtt{A_2} \\ \hline \vdots \\ \hline \mathtt{A_n} \end{bmatrix} \begin{pmatrix} u_x \\ u_y \\ u_z \\ x_0 \\ y_0 \\ z_0 \\ g \end{pmatrix}$$

This then an over-determined equation of the form $\mathtt{A}\mathbf{x} = \mathbf{b}$, and can be solved in MATLAB with the slash operator or the pseudoinverse.

In the second part, when g is known, there are only six unknown parameters, and so your parameter vector will be

$$(u_x \quad u_y \quad u_z \quad x_0 \quad y_0 \quad z_0)^T$$

You will therefore need a different matrix `A` and vector **b**.

2. **Building the matrix `A`**
   Each data point from the text file contributes three equations, and so three lines to both `A` and **b**. The best way to build this matrix and vector is with a `for` loop which steps through the rows of the raw data and, for each row, fills in three rows of `A` and **b**. A row i from the text file ($i = 1 \dots n$) provides information for rows $3(i-1)+1$, $3(i-1)+2$ and $3(i-1)+3$ of this matrix and vector.

3. **The position of the stumps**
   The dimensions of the stumps are given in Figure 12. In addition, the script `stumps.m` will draw a representation of the stumps in a 3D figure window.

D. Evaluation and marking

Write a MATLAB script that contains all the commands necessary for this exercise, so that it displays the required numbers and plots the graphs. Use an `if` statement to print out a message saying whether the ball would have hit the stumps. Show this working script to a demonstrator for marking.

Once you have finished this exercise and had it marked, you have qualified for your full twelve marks. This exercise must be marked by the end of your fourth session. You should now read through the rest of this tutorial and, if you have time left, should attempt Exercise 7.