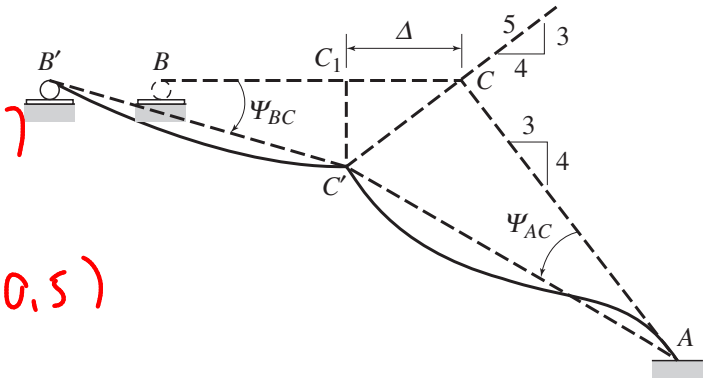
7.

10.28

(a) Fixed end moments:

$$FEM_{BC} = \frac{37.5(4)^2}{12} = 50\,\text{kN}\cdot\text{m}; \quad FEM_{CB} = -50\,\text{kN}\cdot\text{m}$$

(b) Chord rotations:



$$\Psi_{BC} = -\frac{C_1C'}{L_{BC}} = -\frac{(3/4)\Delta}{4} = -0.1875\,\Delta$$

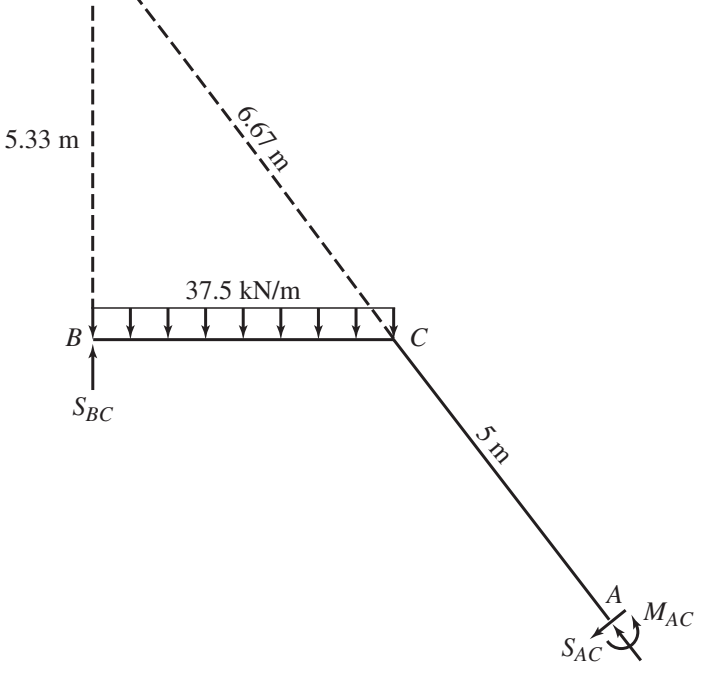
$$\Psi_{AC} = \frac{CC'}{L_{AC}} = \frac{(5/4)\Delta}{5} = 0.25\Delta$$

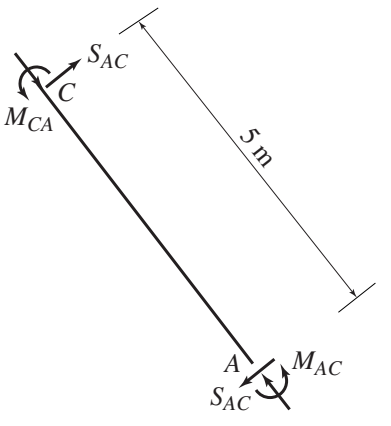
(c) Slope-deflection equations: $M_{BC} = 0$

$$M_{CB} = 0.75\,EI(\theta_C + 0.1875\,\Delta) - 75$$
$$M_{CA} = 0.4\,EI(2\theta_C - 0.75\Delta)$$
$$M_{AC} = 0.4\,EI(\theta_C - 0.75\Delta)$$

Equilibrium equations: $M_{CB} + M_{CA} = 0$

$$1.55\,E\theta_C - 0.1595\,EI\Delta = 75 \qquad (1)$$





15.28 (CONTD.)

$$+\circlearrowright \Sigma M_0 = 0 \quad M_{AC} - S_{AC}(11.67) - (37.5)(4)(2) = 0$$

$$M_{AC} - \left(\frac{M_{AC} + M_{CA}}{5}\right)(11.67) - 300 = 0 \qquad (1)$$

$$-2.4\,EI\theta_C + 1.1\,EI\Delta = 300 \qquad (2)$$

By solving Eqs. (1) and (2), we obtain

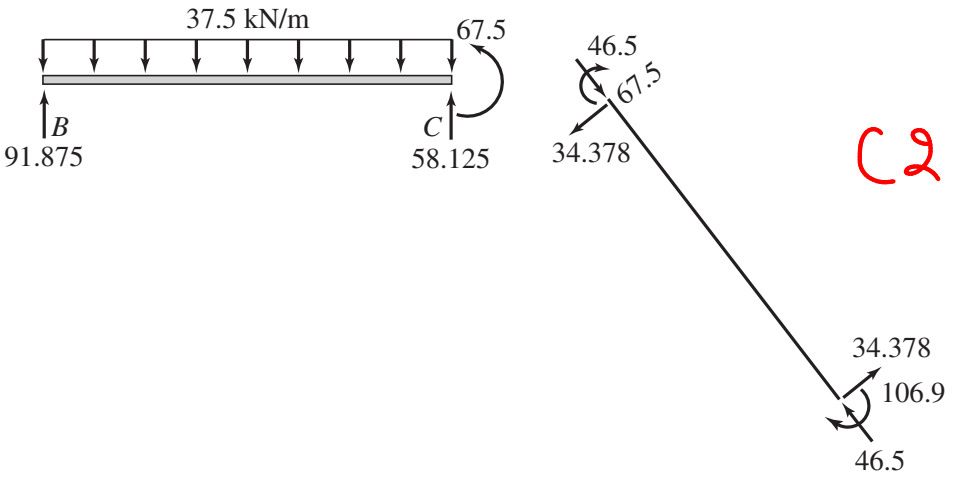
$$EI\theta_C = 98.6\,\text{kN}\cdot\text{m}^2; \quad EI\Delta = 487.84\,\text{kN}\cdot\text{m}^3.$$

Member end moments: Substituting the numerical value of $EI\theta_C$ and $EI\Delta$ into the slope-deflection equations, we obtain:

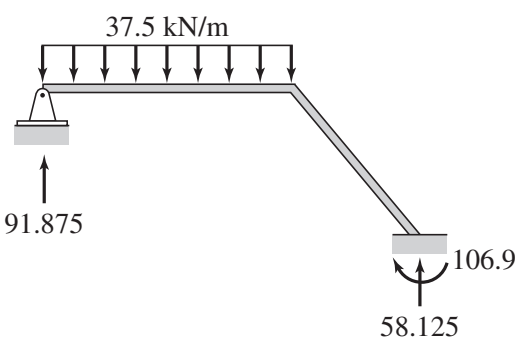
$$M_{CB} = 67.5\,\text{kN}\cdot\text{m}; \quad M_{CA} = -67.5\,\text{kN}\cdot\text{m};$$

$$M_{AC} = -106.9\,\text{kN}\cdot\text{m}$$

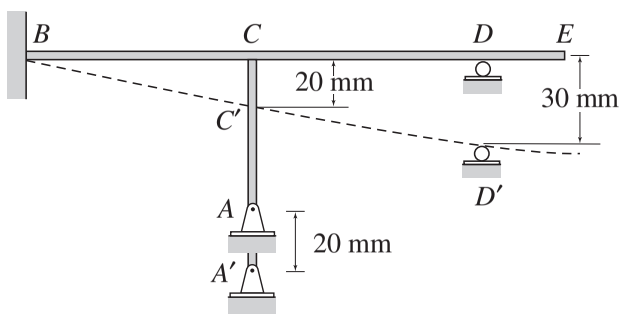
Member end forces



Support reactions



16.23



$$K_{AC} = \frac{3}{4}\left(\frac{I}{4}\right) = \frac{3I}{16}$$

$$K_{BC} = \frac{I}{30};\ K_{CD} = \frac{3}{4}\left(\frac{I}{6}\right) = \frac{I}{8}$$

$$FEM_{AC} = \frac{200(4)}{8} = 100\,\text{kN}\cdot\text{m}; \qquad FEM_{CA} = -100\,\text{kN}\cdot\text{m}$$

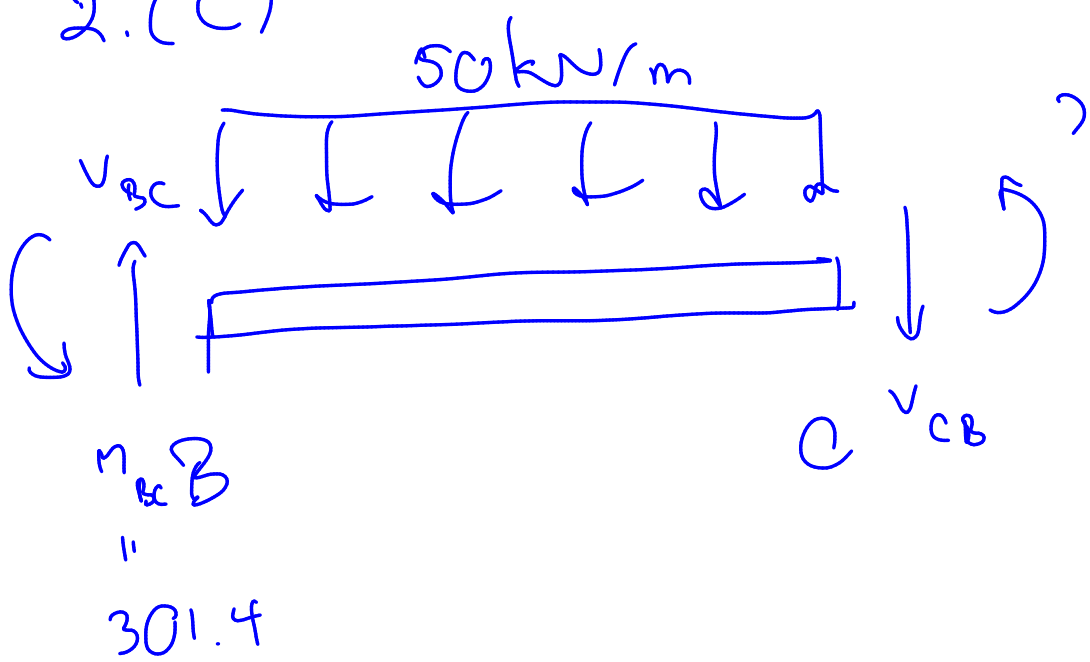
$$FEM_{BC} = \frac{2(30)^2}{12} + \frac{6(70\times10^6)(496\times10^{-6})(0.02)}{(6)^2} = 150 + 115.7 = 265.7\,\text{kN}\cdot\text{m}$$

$$FEM_{CB} = -150 + 115.7 = -34.3\,\text{kN}\cdot\text{m}; \qquad FEM_{CD} = \frac{6(10000)(3000)(0.5)}{(30)^2(12)^3} = 57.9\,\text{kN}\cdot\text{m}$$

$$FEM_{DC} = 57.9\,\text{kN}\cdot\text{m}; \qquad FEM_{DE} = 50(2) = 100\,\text{kN}\cdot\text{m}.$$

BC	*CB*	*CA*	*CD*	*DC*	*DE*	*AC*	
	0.348	0.391	0.261	1	–	1	*DF*
265.7	–34.3	–100	57.9	57.9	100	100	*FEM*
	26.6	29.9	19.9	–157.9		–100	
13.3		–50	–79				
	44.9	50.4	33.7				
22.4							
301.4	37.2	–69.7	32.5	–100	100	0	*Final Moments*

2.(c)



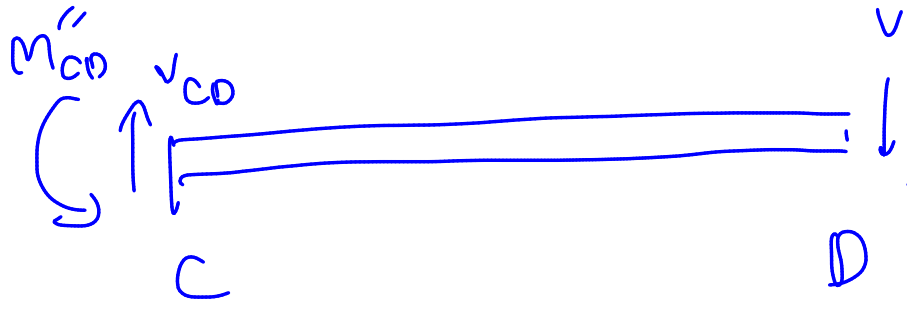
$$V_{BC} = \frac{37.2 + 301.4}{6} + \frac{50 \times 6 \times 3}{6}$$

$$V_{BC} = 206.43 \text{ kN}$$

$$206.43 - 50 \times 6 - V_{CB} = 0$$

$$\therefore V_{CB} = -93.57 \text{ kN}$$

$\frac{32.5}{2}$

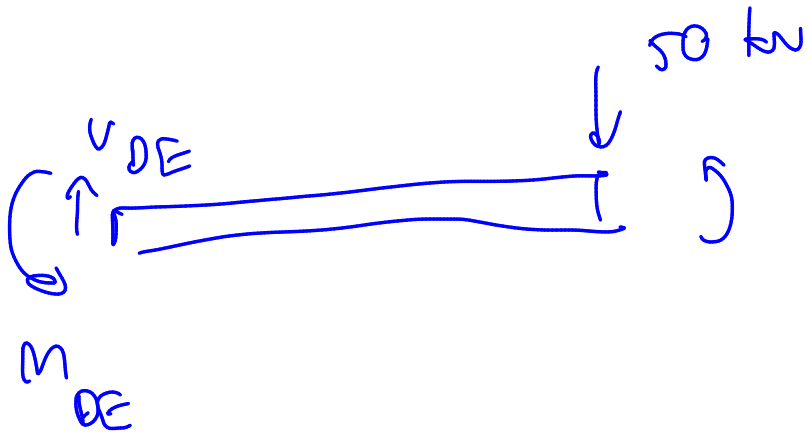


$$V_{CD} = \frac{M_{DC} + M_{CD}}{6}$$

$$= \frac{32.5 - 100}{6}$$

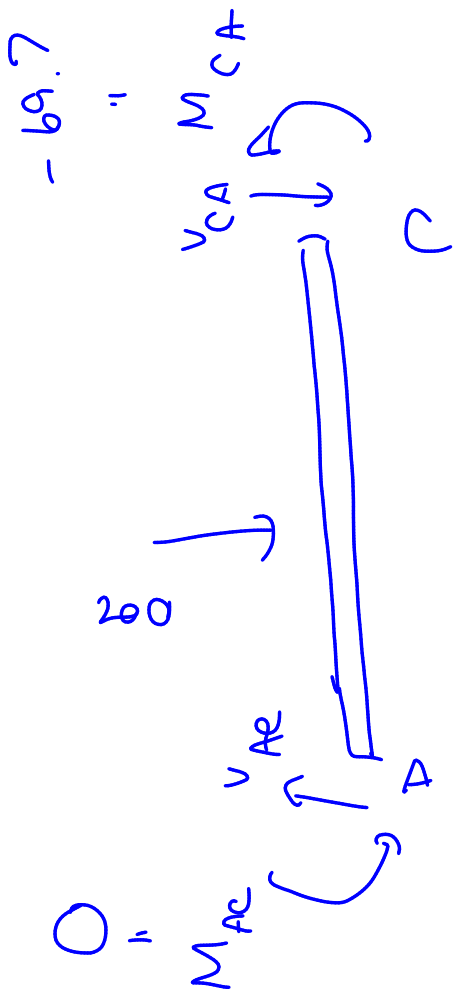
$$= -11.25 \text{ kN}$$

$$V_{DC} = -11.25 \text{ kN}$$



$$M_{DE} = 100 \text{ kN.m}$$

$$V_{DE} = 50 \text{ kN}$$



$$V_{AC} = \frac{M_{CA} + M_{AC}}{4} + \frac{200 \times 2}{4}$$

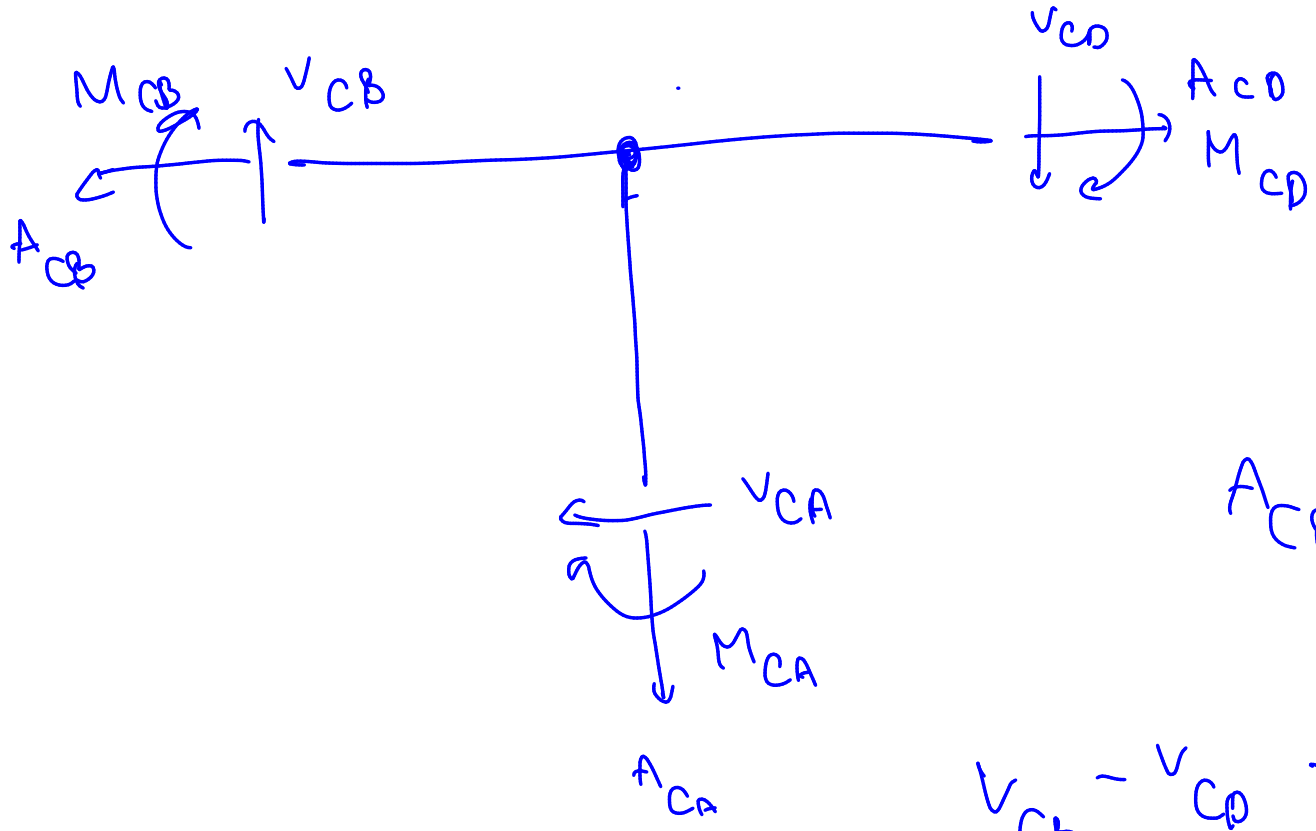
$$V_{AC} = \frac{-69.7 + C}{4} + 100$$

$$= 82.575 \text{ kN}$$

$$82.575 - 200 - V_{CA} = 0$$

$$\therefore V_{CA} = -117.425 \text{ kN}.$$

joint C

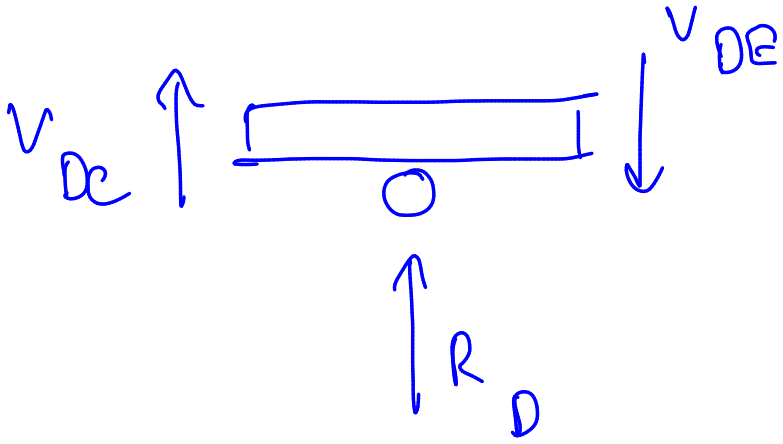


$$A_{CD} = 0$$

$$A_{CB} = -V_{CA}$$
$$= 117.425 \text{ kN}$$

$$V_{CB} - V_{CD} - A_{CA} = 0$$

$$A_{CA} = V_{CB} - V_{CD}$$
$$= -93.57 + 11.25$$
$$= -82.32$$



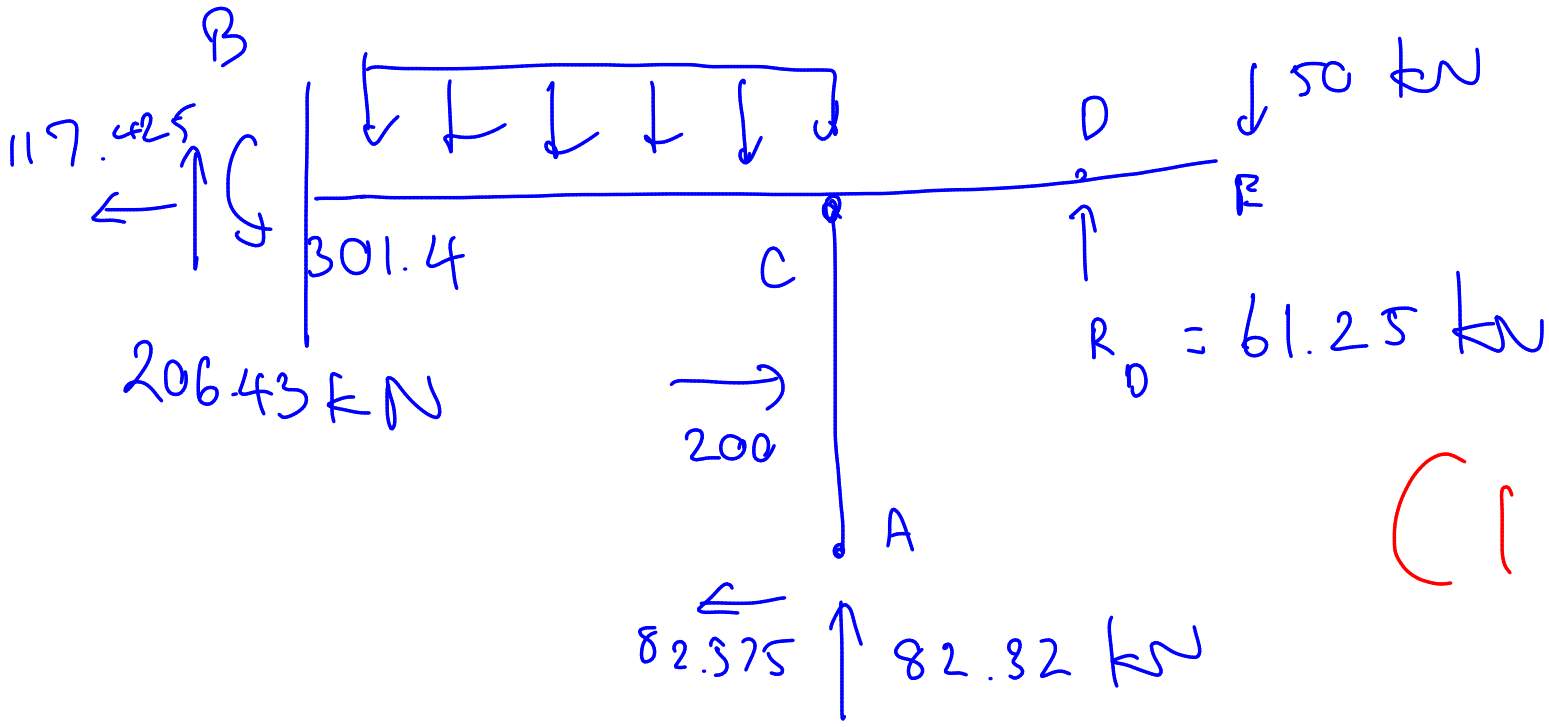
$$R_D + V_{DC} - V_{DE} = 0$$

$$\therefore R_D = V_{DE} - V_{DC}$$

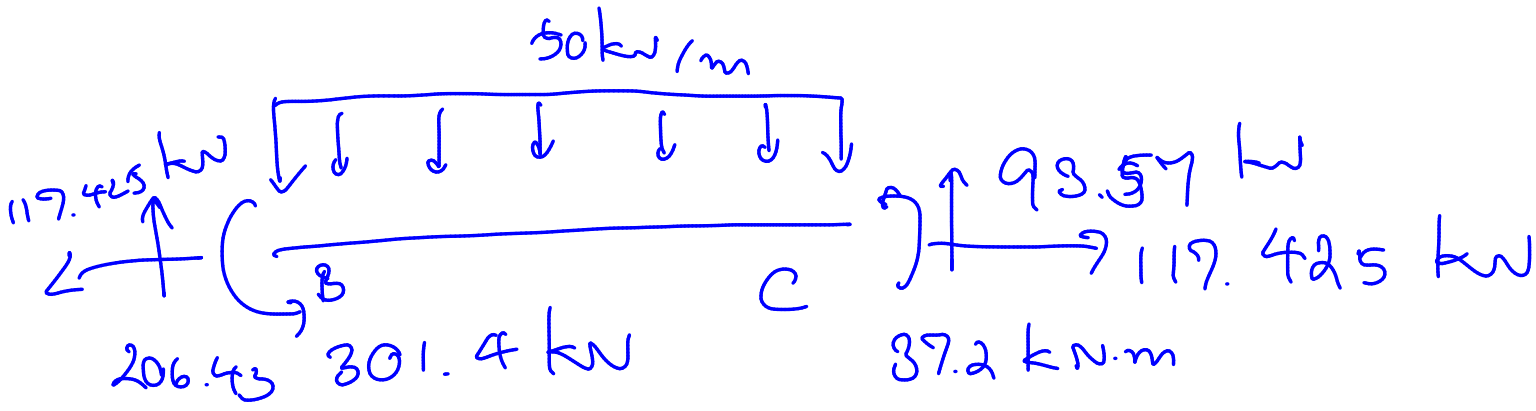
$$= 50 - (-11.25)$$

$$= 61.25$$

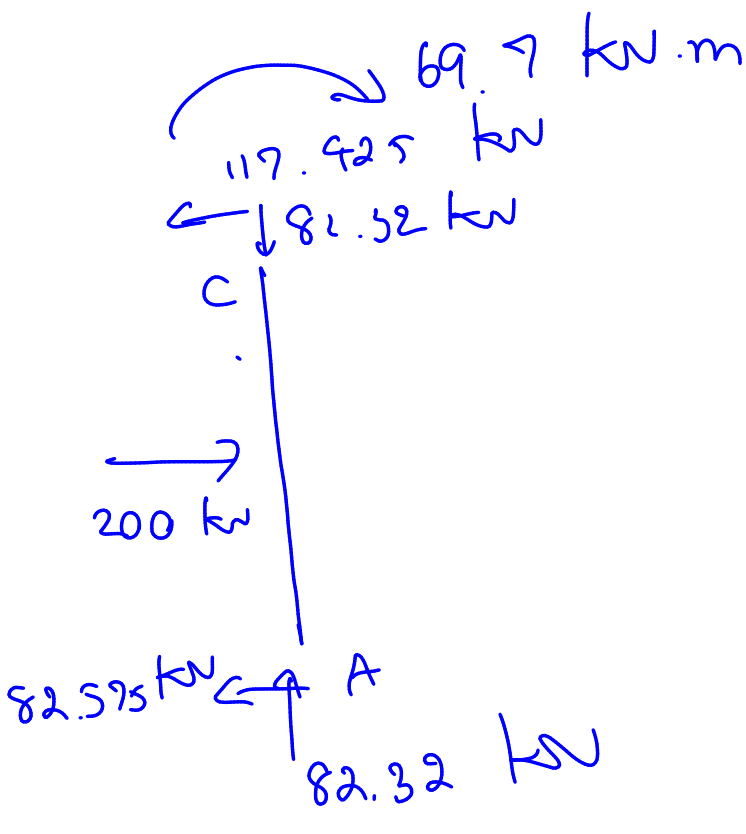
(C)



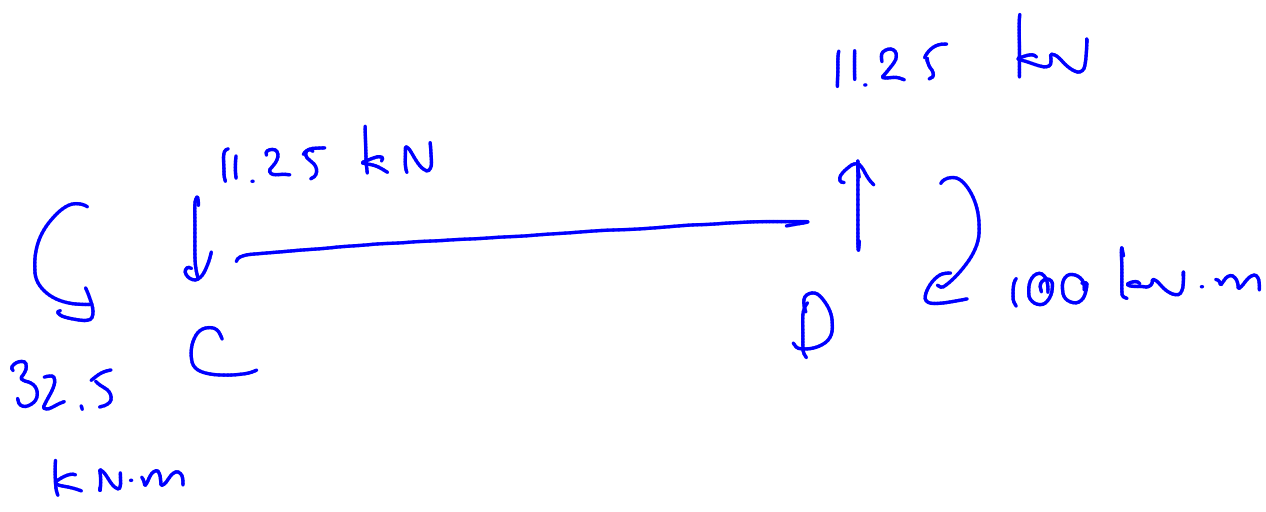
(1 mark)



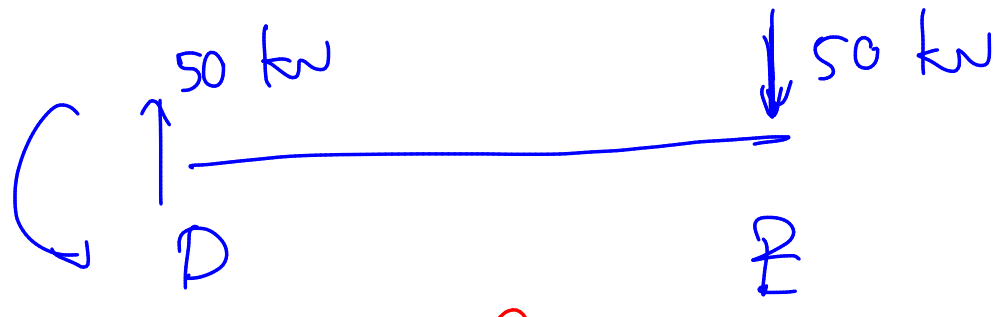
(1)



(1)



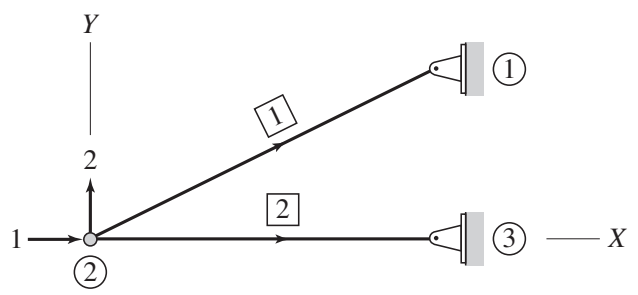
(1)



(1)

ถ้าทำแนวเฉลยได้ไม่เกิน 4

3. (a)



Member 1: $L = 4.47\,\text{m};\quad \cos\theta = 0.894;\quad \sin\theta = 0.447$

$$[K_1] = EA \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0.1788 & 0.0894 & -0.0788 & -0.0894 \\ & 0.0447 & -0.0894 & 0.0447 \\ & \text{sym.} & 0.1788 & 0.0894 \\ & & & 0.0447 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix}$$

(1)

Member 2: $L = 4\,\text{m};\quad \cos\theta = 1;\quad \sin\theta = 0$

$$[K_2] = EA \begin{bmatrix} 1 & 2 & 0 & 0 \\ 0.25 & 0 & -0.25 & 0 \\ & 0 & 0 & 0 \\ & \text{sym.} & 0.25 & 0 \\ & & & 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix}$$

(1)

Structure stiffness matrix:

$$[S] = EA \begin{bmatrix} 1 & 2 \\ 0.4288 & 0.0894 \\ 0.0894 & 0.0447 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

(1)

17.1 (CONTD.)

Joint load vector:

$$\{P\} = \begin{bmatrix} 0 \\ -120 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix} \text{ kN}$$

Joint displacements: By solving the equations

$$\{P\} = [S]\{d\}, \text{ we obtain : } \{d\} = \frac{1}{EA} \begin{bmatrix} 957.87 \\ -4584.07 \end{bmatrix} \begin{matrix} 1 \\ 2 \end{matrix}$$

3(b) & (c)

Member forces:

$$\{v_1\} = \frac{1}{EA} \begin{bmatrix} 957.87 \\ -4584.07 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix} \qquad \{u_1\} = [T_1]\{v_1\} = \frac{1}{EA} \begin{bmatrix} -1197.33 \\ 0 \end{bmatrix}$$

(1.5)

$$\{Q_1\} = [k_1] = \{u_1\} = \begin{bmatrix} -267.89 \\ 267.89 \end{bmatrix} \text{ kN}; \qquad \{F_1\} = [T_1]^T\{Q_1\} = \begin{bmatrix} -240 \\ -120 \\ 240 \\ 120 \end{bmatrix} \text{ kN}$$

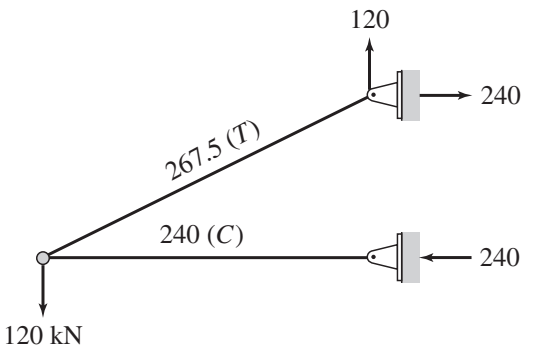
(1)

$$\{v_2\} = \frac{1}{EA} \begin{bmatrix} 958 \\ -4584 \\ 0 \\ 0 \end{bmatrix} \begin{matrix} 1 \\ 2 \\ 0 \\ 0 \end{matrix} \qquad \{u_2\} = [T_2]\{v_2\} = \frac{1}{EA} \begin{bmatrix} 958 \\ 0 \end{bmatrix}$$

(1.5)

$$\{Q_2\} = [k_2]\{u_2\} = \begin{bmatrix} 240 \\ -240 \end{bmatrix} \text{ kN}; \qquad \{F_2\} = [T_2]^T\{Q_2\} = \begin{bmatrix} 240 \\ 0 \\ -240 \\ 0 \end{bmatrix} \text{ k}$$

(1)



(2)