Slope-deflection method

Slope-deflection equation

$$M_{nf} = \frac{2EI}{L}\left(2\theta_n + \theta_f \text{-} 3\psi\right) + FEM_{nf}$$

Modified slope-deflection equation

$$M_{rh} = \frac{3EI}{L}(\theta_r \text{-} \psi) + \left(FEM_{rh} \text{-} \frac{FEM_{hr}}{2}\right)$$

Sidesway degrees of freedom of a frame

$$ss = 2j - [2(f+h) + r + m]$$

Moment Distribution Method

Hinged at the near end and fixed at the far end

$$M = \left(\frac{4EI}{L}\right)\theta$$

$$\bar{K} = \frac{4EI}{L}$$

$$K = \frac{\bar{K}}{4E} = \frac{I}{L}$$

The far end hinged

$$M = \left(\frac{3EI}{L}\right)\theta$$

$$\bar{K} = \frac{3EI}{L}$$

Carryover factor

$$M_{BA} = \begin{cases} \dfrac{M}{2} & \text{if far end of member is fixed} \\ 0 & \text{if far end of member is hinged} \end{cases}$$

Distribution factor

$$DF = \frac{K}{\sum K}$$

Frames with sidesway

$$M = M_O + M_R = \mathrm{M_O} + \left(\frac{R}{Q}\right) M_Q$$

Matrix Structural Analysis

$$\mathbf{P}\text{-}\mathbf{P_f} = \boldsymbol{Sd}$$

$$\mathbf{Q} = \mathbf{ku} + \mathbf{Q}_f$$

$$\mathbf{Q} = \mathbf{TF}$$

$$\mathbf{u} = \mathbf{Tv}$$

$$\mathbf{F} = \mathbf{T}^T\mathbf{Q}$$

$$\mathbf{v} = \mathbf{T}^T\mathbf{u}$$

$$\mathbf{F} = \mathrm{K}\mathbf{v} + \mathbf{F}_f$$

Frame

$$\mathbf{k} = \frac{EI}{L^3}\begin{bmatrix} \frac{AL^2}{I} & 0 & 0 & -\frac{AL^2}{I} & 0 & 0 \\ 0 & 12 & 6L & 0 & -12 & 6L \\ 0 & 6L & 4L^2 & 0 & -6L & 2L^2 \\ \frac{AL^2}{I} & 0 & 0 & \frac{AL^2}{I} & 0 & 0 \\ 0 & -12 & -6L & 0 & 12 & -6L \\ 0 & 6L & 2L^2 & 0 & -6L & 4L^2 \end{bmatrix}$$

$$\mathbf{T} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 & 0 & 0 \\ -\sin\theta & \cos\theta & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & \cos\theta & \sin\theta & 0 \\ 0 & 0 & 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}$$

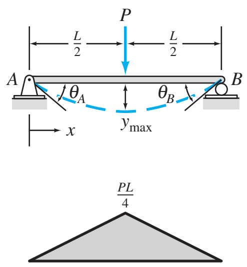
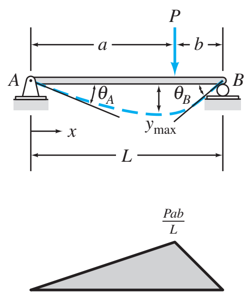
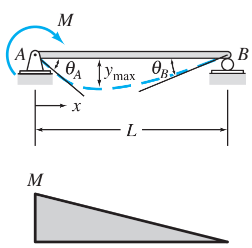
Beam

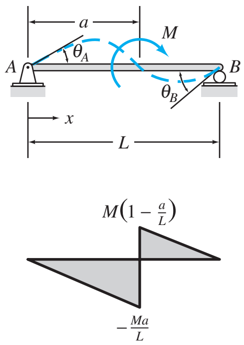
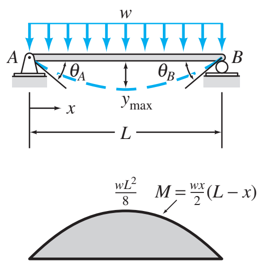
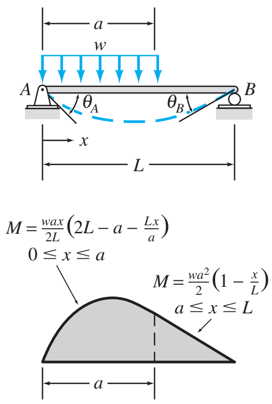
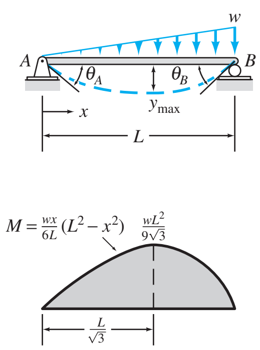
$$\mathbf{k} = \frac{EI}{L^3}\begin{bmatrix} 12 & 6L & -12 & 6L \\ 6L & 4L^2 & -6L & 2L^2 \\ -12 & -6L & 12 & -6L \\ 6L & 2L^2 & -6L & 4L^2 \end{bmatrix}$$

Truss

$$\mathbf{k} = \frac{EA}{L}\begin{bmatrix} 1 & -1 \\ -1 & 1 \end{bmatrix}$$

$$\mathbf{T} = \begin{bmatrix} \cos\theta & \sin\theta & 0 & 0 \\ 0 & 0 & \cos\theta & \sin\theta \end{bmatrix}$$

Beam, Loading, and Bending Moment Diagram	Equations for Slope and Deflection
	$0 \le x \le \frac{L}{2}$: $\theta = \frac{P}{16EI}(4x^2 - L^2)$ $y = \frac{P}{48EI}(4x^3 - 3L^2x)$ $\theta_A = -\frac{PL^2}{16EI}$; $\theta_B = \frac{PL^2}{16EI}$ $y_{\max} = -\frac{PL^3}{48EI}$
	$0 \le x \le a$: $\theta = \frac{Pb}{6EIL}(3x^2 + b^2 - L^2)$ $y = \frac{Pb}{6EIL}(x^3 + b^2x - L^2x)$ $a \le x \le L$: $\theta = \frac{Pa}{6EIL}[L^2 - a^2 - 3(L-x)^2]$ $y = \frac{Pa(L-x)}{6EIL}(x^2 + a^2 - 2Lx)$ $\theta_A = -\frac{Pb}{6EIL}(L^2 - b^2)$ $\theta_B = \frac{Pa}{6EIL}(L^2 - a^2)$ For $a \ge b$: $y_{\max} = -\frac{Pb}{9\sqrt{3}EIL}(L^2 - b^2)^{3/2}$ at $x = \left(\frac{L^2 - b^2}{3}\right)^{1/2}$
	$\theta = -\frac{M}{6EIL}(3x^2 - 6Lx + 2L^2)$ $y = -\frac{M}{6EIL}(x^3 - 3Lx^2 + 2L^2x)$ $\theta_A = -\frac{ML}{3EI}$; $\theta_B = \frac{ML}{6EI}$ $y_{\max} = -\frac{ML^2}{9\sqrt{3}EI}$ at $x = L\left(1 - \frac{1}{\sqrt{3}}\right)$

Beam, Loading, and Bending Moment Diagram	Equations for Slope and Deflection
	$0 \le x \le a$: $\theta = \frac{M}{6EIL}(-3x^2 + 6aL - 3a^2 - 2L^2)$ $y = \frac{M}{6EIL}(-x^3 + 6aLx - 3a^2x - 2L^2x)$ $\theta_A = \frac{M}{6EIL}(6aL - 3a^2 - 2L^2)$ $\theta_B = \frac{M}{6EIL}(L^2 - 3a^2)$
	$\theta = -\frac{w}{24EI}(4x^3 - 6Lx^2 + L^3)$ $y = -\frac{w}{24EI}(x^4 - 2Lx^3 + L^3x)$ $\theta_A = -\frac{wL^3}{24EI}$ $\theta_B = \frac{wL^3}{24EI}$ $y_{max} = -\frac{5wL^4}{384EI}$ at $x = \frac{L}{2}$
	$0 \le x \le a$: $\theta = -\frac{w}{24EIL}[4Lx^3 - 6a(2L - a)x^2 + a^2(2L - a)^2]$ $y = -\frac{w}{24EIL}[Lx^4 - 2a(2L - a)x^3 + a^2(2L - a)^2x]$ $a \le x \le L$: $\theta = -\frac{wa^2}{24EIL}(6x^2 - 12Lx + a^2 + 4L^2)$ $y = -\frac{wa^2}{24EIL}(L - x)(-2x^2 + 4Lx - a^2)$ $\theta_A = -\frac{wa^2}{24EIL}(2L - a)^2$ $\theta_B = \frac{wa^2}{24EIL}(2L^2 - a^2)$
	$\theta = -\frac{w}{360EIL}(15x^4 - 30L^2x^2 + 7L^4)$ $y = -\frac{w}{360EIL}(3x^5 - 10L^2x^3 + 7L^4x)$ $\theta_A = -\frac{7wL^3}{360EI}$ $\theta_B = \frac{wL^3}{45EI}$ $y_{max} = -0.00652\frac{wL^4}{EI}$ at $x = 0.5193L$

FIXED-END MOMENTS