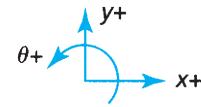
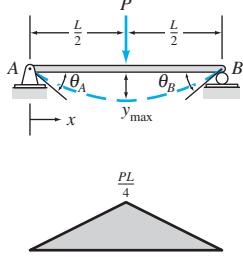
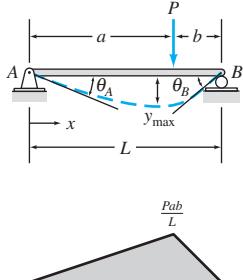
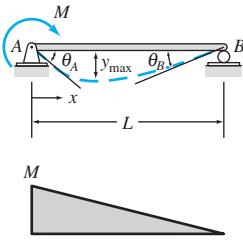
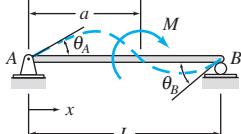
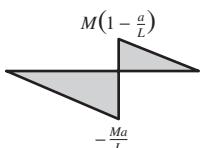
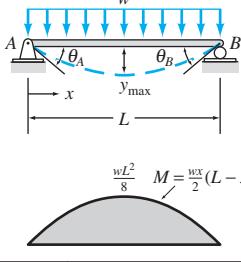
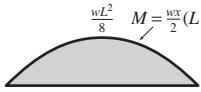
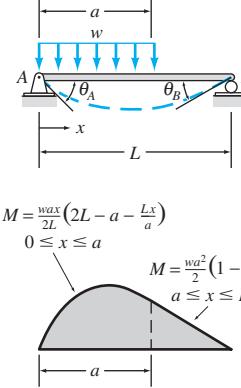
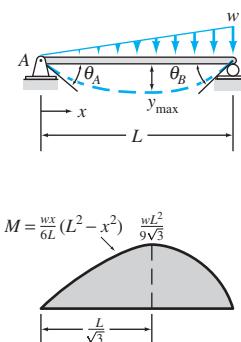


## BENDING MOMENTS, SLOPES, AND DEFLECTIONS OF BEAMS UNDER VARIOUS LOADING CONDITIONS



Beam, Loading, and Bending Moment Diagram	Equations for Slope and Deflection
	$0 \leq x \leq a :$ $\theta = \frac{P}{2EI}(x^2 - 2ax)$ $y = \frac{P}{6EI}(x^3 - 3ax^2)$ $a \leq x \leq L :$ $\theta = -\frac{Pa^2}{2EI}$ $y = \frac{Pa^2}{6EI}(a - 3x)$ $\theta_B = -\frac{Pa^2}{2EI}; \quad y_B = -\frac{Pa^2}{6EI}(3L - a)$
	$0 \leq x \leq a :$ $\theta = -\frac{Mx}{EI}$ $y = -\frac{Mx^2}{2EI}$ $a \leq x \leq L :$ $\theta = -\frac{Ma}{EI}$ $y = \frac{Ma}{2EI}(a - 2x)$ $\theta_B = -\frac{Ma}{EI}; \quad y_B = -\frac{Ma}{2EI}(2L - a)$
	$0 \leq x \leq a :$ $\theta = \frac{w}{6EI}(3ax^2 - 3a^2x - x^3)$ $y = \frac{w}{24EI}(4ax^3 - 6a^2x^2 - x^4)$ $a \leq x \leq L :$ $\theta = -\frac{wa^3}{6EI}$ $y = \frac{wa^3}{24EI}(a - 4x)$ $\theta_B = -\frac{wa^3}{6EI}; \quad y_B = -\frac{wa^3}{24EI}(4L - a)$
	$0 \leq x \leq a :$ $\theta = \frac{w}{24EIa}(x^4 - 4ax^3 + 6a^2x^2 - 4a^3x)$ $y = \frac{w}{120EIa}(x^5 - 5ax^4 + 10a^2x^3 - 10a^3x^2)$ $a \leq x \leq L :$ $\theta = -\frac{wa^3}{24EI}$ $y = \frac{wa^3}{120EI}(-5x + a)$ $\theta_B = -\frac{wa^3}{24EI}; \quad y_B = -\frac{wa^3}{120EI}(5L - a)$

Beam, Loading, and Bending Moment Diagram	Equations for Slope and Deflection
 	$0 \leq x \leq \frac{L}{2} :$ $\theta = \frac{P}{16EI}(4x^2 - L^2)$ $y = \frac{P}{48EI}(4x^3 - 3L^2x)$ $\theta_A = -\frac{PL^2}{16EI}; \quad \theta_B = \frac{PL^2}{16EI}$ $y_{\max} = -\frac{PL^3}{48EI}$
 	$0 \leq x \leq a :$ $\theta = \frac{Pb}{6EI_L}(3x^2 + b^2 - L^2)$ $y = \frac{Pb}{6EI_L}(x^3 + b^2x - L^2x)$  $a \leq x \leq L :$ $\theta = \frac{Pa}{6EI_L}[L^2 - a^2 - 3(L-x)^2]$ $y = \frac{Pa(L-x)}{6EI_L}(x^2 + a^2 - 2Lx)$ $\theta_A = -\frac{Pb}{6EI_L}(L^2 - b^2)$ $\theta_B = \frac{Pa}{6EI_L}(L^2 - a^2)$
 	$\theta = -\frac{M}{6EI_L}(3x^2 - 6Lx + 2L^2)$ $y = -\frac{M}{6EI_L}(x^3 - 3Lx^2 + 2L^2x)$ $\theta_A = -\frac{ML}{3EI}; \quad \theta_B = \frac{ML}{6EI}$ $y_{\max} = -\frac{ML^2}{9\sqrt{3}EI}$ at $x = L \left(1 - \frac{1}{\sqrt{3}}\right)$

Beam, Loading, and Bending Moment Diagram	Equations for Slope and Deflection
 	$0 \leq x \leq a :$ $\theta = \frac{M}{6EI}(-3x^2 + 6aL - 3a^2 - 2L^2)$ $y = \frac{M}{6EI}(-x^3 + 6aLx - 3a^2x - 2L^2x)$ $\theta_A = \frac{M}{6EI}(6aL - 3a^2 - 2L^2)$ $\theta_B = \frac{M}{6EI}(L^2 - 3a^2)$
 	$\theta = -\frac{w}{24EI}(4x^3 - 6Lx^2 + L^3)$ $y = -\frac{w}{24EI}(x^4 - 2Lx^3 + L^3x)$ $\theta_A = -\frac{wL^3}{24EI}$ $\theta_B = \frac{wL^3}{24EI}$ $y_{\max} = -\frac{5wL^4}{384EI} \text{ at } x = \frac{L}{2}$
 $M = \frac{wx}{2L}(2L - a - \frac{Lx}{a}) \quad 0 \leq x \leq a$ $M = \frac{wa^2}{2}(1 - \frac{x}{L}) \quad a \leq x \leq L$	$0 \leq x \leq a :$ $\theta = -\frac{w}{24EI}[4Lx^3 - 6a(2L - a)x^2 + a^2(2L - a)^2]$ $y = -\frac{w}{24EI}[Lx^4 - 2a(2L - a)x^3 + a^2(2L - a)^2x]$ $a \leq x \leq L :$ $\theta = -\frac{wa^2}{24EI}(6x^2 - 12Lx + a^2 + 4L^2)$ $y = -\frac{wa^2}{24EI}(L - x)(-2x^2 + 4Lx - a^2)$ $\theta_A = -\frac{wa^2}{24EI}(2L - a)^2$ $\theta_B = \frac{wa^2}{24EI}(2L^2 - a^2)$
 $M = \frac{wx}{6L}(L^2 - x^2) \quad 0 \leq x \leq a$ $M = \frac{wL^2}{9\sqrt{3}} \quad a \leq x \leq L$	$\theta = -\frac{w}{360EI}(15x^4 - 30L^2x^2 + 7L^4)$ $y = -\frac{w}{360EI}(3x^5 - 10L^2x^3 + 7L^4x)$ $\theta_A = -\frac{7wL^3}{360EI}$ $\theta_B = \frac{wL^3}{45EI}$ $y_{\max} = -0.00652 \frac{wL^4}{EI} \text{ at } x = 0.5193L$

## FIXED-END MOMENTS

