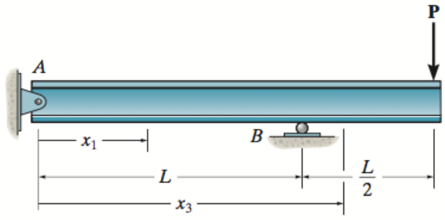
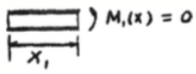
***12–4.** Determine the equations of the elastic curve using the x_1 and x_2 coordinates. EI is constant.



$$EI\frac{d^2v_1}{dx_1{}^2} = M_1(x)$$

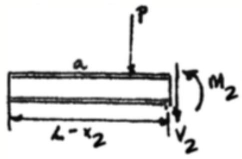
$$M_1(x) = 0; \qquad EI\frac{d^2v_1}{dx_1{}^2} = 0$$



$$EI\frac{dv_1}{dx_1} = C_1 \tag{1}$$

$$EI\,v_1 = C_1x_1 + C_2 \tag{2}$$

$$M_2(x) = Px_2 - P(L-a)$$



$$EI\frac{d^2v_2}{dx_2{}^2} = Px_2 - P(L-a)$$

$$EI\frac{dv_2}{dx_2} = \frac{P}{2}x_2{}^2 - P(L-a)x_2 + C_3 \tag{3}$$

$$EI\,v_2 = \frac{P}{6}x_2{}^3 - \frac{P(L-a)x_2^2}{2} + C_3x_2 + C_4 \tag{4}$$

Boundary conditions:

At $x_2 = 0$, $\frac{dv_2}{dx_2} = 0$

From Eq. (3), $0 = C_3$

At $x_2 = 0$, $v_2 = 0$

$0 = C_4$

Continuity condition:

At $x_1 = a, x_2 = L - a;\qquad \frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$

From Eqs. (1) and (3),

$$C_1 = -\left[\frac{P(L-a)^2}{2} - P(L-a)^2\right]; \qquad C_1 = \frac{P(L-a)^2}{2}$$

At $x_1 = a, x_2 = L - a, v_1 = v_2$

From Eqs. (2) and (4),

$$\left(\frac{P(L-a)^2}{2}\right)a + C_2 = \frac{P(L-a)^3}{6} - \frac{P(L-a)^3}{2}$$

$$C_2 = -\frac{Pa(L-a)^2}{2} - \frac{P(L-a)^3}{3}$$

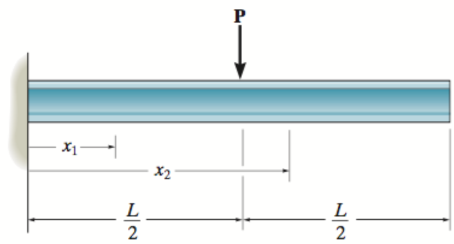
From Eq. (2),

$$v_1 = \frac{P}{6EI}[3(L-a)^2x_1 - 3a(L-a)^2 - 2(L-a)^3] \qquad \textbf{Ans.}$$

For Eq. (4),

$$v_2 = \frac{P}{6EI}[x_2^2 - 3(L-a)x_3^3] \qquad \textbf{Ans.}$$

***12–8.** Determine the equations of the elastic curve for the beam using the x_1 and x_2 coordinates. EI is constant.



Referring to the FBDs of the beam's cut segments shown in Fig. *b* and *c*,

$$\circlearrowleft + \Sigma M_O = 0; \qquad M(x_1) + \frac{PL}{2} - Px_1 = 0 \qquad M(x_1) = Px_1 - \frac{PL}{2}$$

And

$$\circlearrowleft + \Sigma M_O = 0; \qquad M(x_2) = 0$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1^2} = Px_1 - \frac{PL}{2}$$

$$EI\frac{dv_1}{dx_1} = \frac{P}{2}x_1{}^2 - \frac{PL}{2}x_1 + C_1 \qquad (1)$$

$$EI\,v_1 = \frac{P}{6}x_1{}^3 - \frac{PL}{4}x_1{}^2 + C_1x_1 + C_2 \qquad (2)$$

For coordinate x_2,

$$EI\frac{d^2v_2}{dx_2^2} = 0$$

$$EI\frac{dv_2}{dx_2} = C_3 \qquad (3)$$

$$EI\,v_2 = C_3x_2 = C_4 \qquad (4)$$

At $x_1 = 0, \frac{dv_1}{dx_1} = 0$. Then, Eq.(1) gives

$$EI(0) = \frac{P}{2}(0^2) - \frac{PL}{2}(0) + C_1 \qquad C_1 = 0$$

At $x_1 = 0, v_1 = 0$. Then, Eq(2) gives

$$EI(0) = \frac{P}{6}(0^3) - \frac{PL}{4}(0^2) + 0 + C_2 \qquad C_2 = 0$$

At $x_1 = x_2 = \frac{L}{2}, \frac{dv_1}{dx_1} = \frac{dv_2}{dx_2}$. Thus, Eqs.(1) and (3) gives

$$\frac{P}{2}\left(\frac{L}{2}\right)^2 - \frac{PL}{2}\left(\frac{L}{2}\right) = C_3 \qquad C_3 = -\frac{PL^2}{8}$$

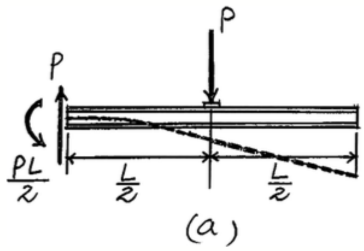
Also, at $x_1 = x_2 = \frac{L}{2}, v_1 = v_2$. Thus, Eqs, (2) and (4) gives

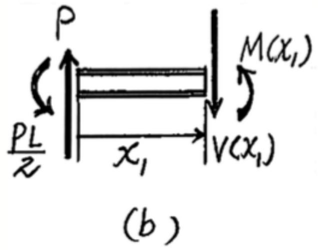
$$\frac{P}{6}\left(\frac{L}{2}\right)^3 - \frac{PL}{4}\left(\frac{L}{2}\right)^2 = \left(-\frac{PL^2}{8}\right)\left(\frac{L}{2}\right) + C_4 \qquad C_4 = \frac{PL^3}{48}$$

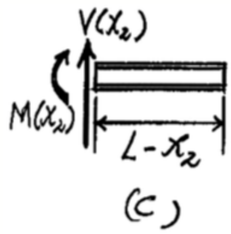
Substitute the values of C_1 and C_2 into Eq. (2) and C_3 and C_4 into Eq (4),

$$v_1 = \frac{P}{12EI}\left(2x_1^3 - 3Lx_1^2\right) \qquad \textbf{Ans.}$$

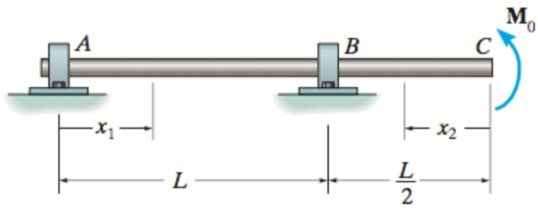
$$v_2 = \frac{PL^2}{48EI}(-6x_2 + L) \qquad \textbf{Ans.}$$







•**12–17.** Determine the equations of the elastic curve for the shaft using the x_1 and x_2 coordinates. Specify the slope at A and the deflection at C. EI is constant.



Referring to the FBDs of the shaft's cut segments shown in Fig. *b* and *c*,

$$\zeta + \Sigma M_O = 0; \qquad M(x_1) - \frac{M_O}{L}x_1 = 0 \qquad M(x_1) = \frac{M_O}{L}x_1$$

And

$$\zeta + \Sigma M_O = 0; \qquad M_O - M(x_2) = 0 \qquad M(x_2) = M_O$$

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1{}^2} = \frac{M_O}{L}x_1$$

$$EI\frac{dv_1}{dx_1} = \frac{M_O}{2L}x_1{}^2 + C_1 \qquad (1)$$

$$EI\,v_1 = \frac{M_O}{6L}x_1{}^3 + C_1x_1 + C_2 \qquad (2)$$

For coordinate x_2,

$$EI\frac{d^2v_2}{dx_2{}^2} = M_O$$

$$EI\frac{dv_2}{dx_2} = M_Ox_2 + C_3 \qquad (3)$$

$$EI\,v_2 = \frac{M_O}{2}x_2{}^2 + C_3x_2 + C_4 \qquad (4)$$

At $x_1 = 0, v_1 = 0$. Then, Eq. (2) gives

$$EI\,(0) = \frac{M_O}{6L}(0^3) + C_1(0) + C_2 \qquad C_2 = 0$$

At $x_1 = L, v_1 = 0$. Then, Eq. (2) gives

$$C_1 = \frac{-ML}{6}$$

Also, at $x_2 = \frac{L}{2}, v_2 = 0$. Then Eq. (4) gives.

$$EI(0) = \frac{M_O}{2}\left(\frac{L}{2}\right)^2 + C_3\left(\frac{L}{2}\right) + C_4$$

$$C_3L + 2C_4 = -\frac{M_OL^2}{4} \qquad (5)$$

•12–17. Continued

At $x_1 = L$ and $x_2 = \frac{L}{2}, \frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. Then, Eq. (1) and (3) give

$$\frac{M_O}{2L}\left(L^2\right) - \frac{M_O L}{6} = -\left[M_O\left(\frac{L}{2}\right) + C_3\right] \qquad C_3 = -\frac{5M_O L}{6}$$

Substitute the result of C_3 into Eq. (5),

$$C_4 = \frac{7M_0 L^2}{24}$$

Substitute the value of C_1 into Eq. (1),

$$\frac{dv_1}{dx_1} = \frac{M_O}{6LEI}\left(3{x_1}^2 - L^2\right)$$

At A, $x_1 = 0$. Thus

$$\theta_A = \frac{dv_1}{dx_1}\bigg|_{x_1=0} = -\frac{M_O}{6EI} = \frac{M_O L}{6EI} \qquad \textbf{Ans.}$$

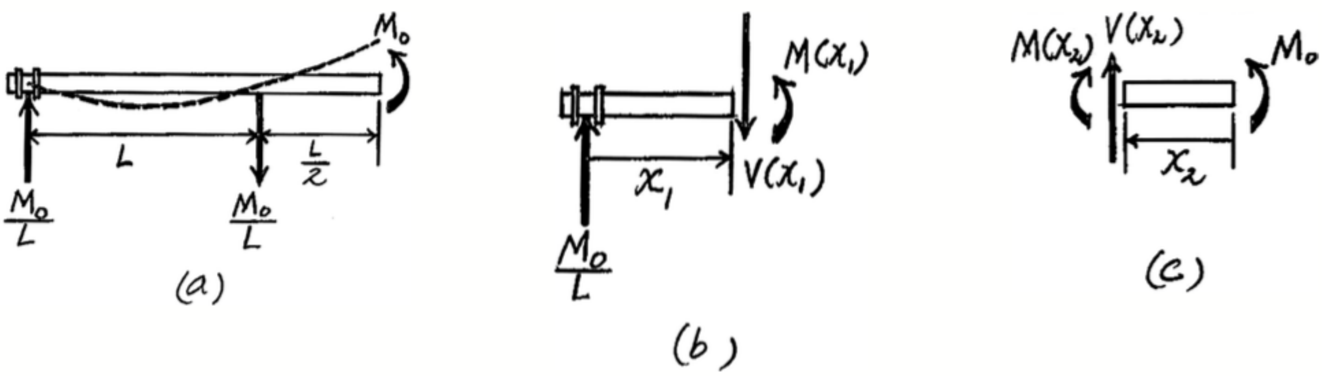
Substitute the values of C_1 and C_2 into Eq (2) and C_3 and C_4 into Eq. (4),

$$v_1 = \frac{M_O}{6EIL}\left({x_1}^3 - L^2 x_1\right) \qquad \textbf{Ans.}$$

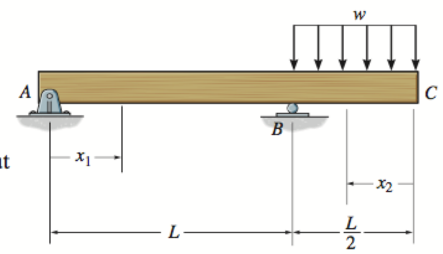
$$v_2 = \frac{M_O}{24EI}\left(12{x_2}^2 - 20\,Lx_2 + 7L^2\right) \qquad \textbf{Ans.}$$

At C, $x_2 = 0$. Thus

$$v_C = v_2\bigg|_{x_2=0} = \frac{7M_O L^2}{24EI} \quad \uparrow \qquad \textbf{Ans.}$$



•**12–21.** Determine the elastic curve in terms of the and coordinates and the deflection of end C of the overhang beam. EI is constant.



Support Reactions and Elastic Curve. As shown in Fig. *a*.

Moment Functions. Referring to the free-body diagrams of the beam's cut segments, Fig. *b*, $M(x_1)$ is

$$\circlearrowleft + \Sigma M_O = 0; \qquad M(x_1) + \frac{wL}{8}x_1 = 0 \qquad M(x_1) = -\frac{wL}{8}x_1$$

and $M(x_2)$ is

$$\circlearrowleft + \Sigma M_O = 0; \qquad -M(x_2) - wx_2\left(\frac{x_2}{2}\right) = 0 \qquad M(x_2) = -\frac{w}{2}x_2^{\,2}$$

Equations of Slope and Elastic Curve.

$$EI\frac{d^2v}{dx^2} = M(x)$$

For coordinate x_1,

$$EI\frac{d^2v_1}{dx_1^{\,2}} = -\frac{wL}{8}x_1$$

$$EI\frac{dv_1}{dx_1} = -\frac{wL}{16}x_1^{\,2} + C_1 \qquad (1)$$

$$EIv_1 = -\frac{wL}{48}x_1^{\,3} + C_1x_1 + C_2 \qquad (2)$$

For coordinate x_2,

$$EI\frac{d^2v_2}{dx_2^{\,2}} = -\frac{w}{2}x_2^{\,2}$$

$$EI\frac{dv_2}{dx_2} = -\frac{w}{6}x_2^{\,3} + C_3 \qquad (3)$$

$$EIv_2 = -\frac{w}{24}x_2^{\,4} + C_3x_2 + C_4 \qquad (4)$$

Boundary Conditions. At $x_1 = 0, v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{wL}{48}\left(0^3\right) + C_1(0) + C_2 \qquad C_2 = 0$$

At $x_1 = L, v_1 = 0$. Then, Eq. (2) gives

$$EI(0) = -\frac{wL}{48}\left(L^3\right) + C_1L + 0 \qquad C_1 = \frac{wL^3}{48}$$

At $x_2 = \frac{L}{2}, v_2 = 0$. Then, Eq. (4) gives

$$EI(0) = -\frac{w}{24}\left(\frac{L}{2}\right)^4 + C_3\left(\frac{L}{2}\right) + C_4$$

$$\frac{L}{2}C_3 + C_4 = \frac{wL^4}{384} \qquad (5)$$

•12–21. Continued

Continuity Conditions. At $x_1 = L$ Land $x_2 = \frac{L}{2}, \frac{dv_1}{dx_1} = -\frac{dv_2}{dx_2}$. Thus, Eqs. (1) and (3) give

$$-\frac{wL}{16}\left(L^2\right) + \frac{wL^3}{48} = -\left[-\frac{w}{6}\left(\frac{L}{2}\right)^3 + C_3\right] \qquad C_3 = \frac{wL^3}{16}$$

Substituting the value of C_3 into Eq. (5),

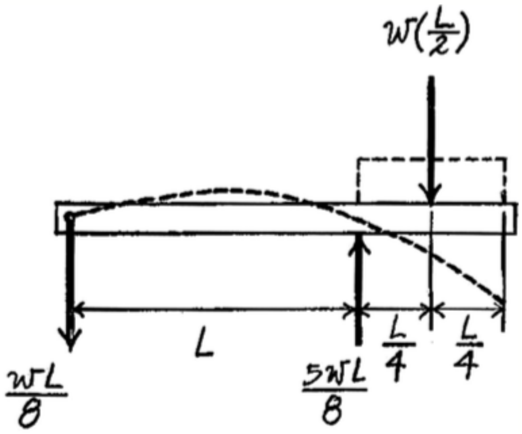
$$C_4 = -\frac{11wL^4}{384}$$

Substituting the values of C_3 and C_4 into Eq. (4),

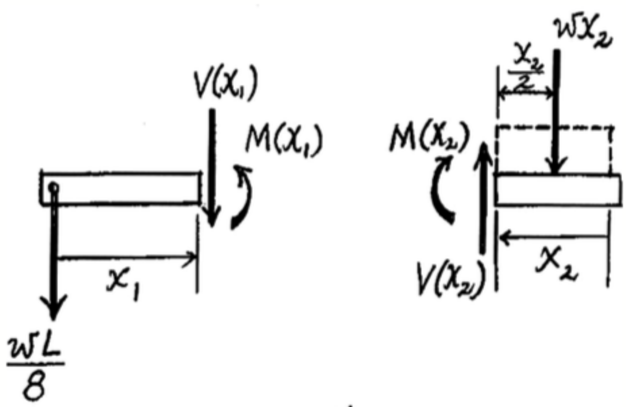
$$v_2 = \frac{w}{384EI}\left(-16x_2{}^4 + 24L^3x_2 - 11L^4\right)$$

At C, $x_2 = 0$. Thus,

$$v_C = v_2\big|_{x_2=0} = -\frac{11wL^4}{384EI} = \frac{11wL^4}{384EI} \downarrow$$ **Ans.**



(a)



(b)