CE 221: MECHANICS OF SOLIDS I

CHAPTER 2: STRAIN

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Outline

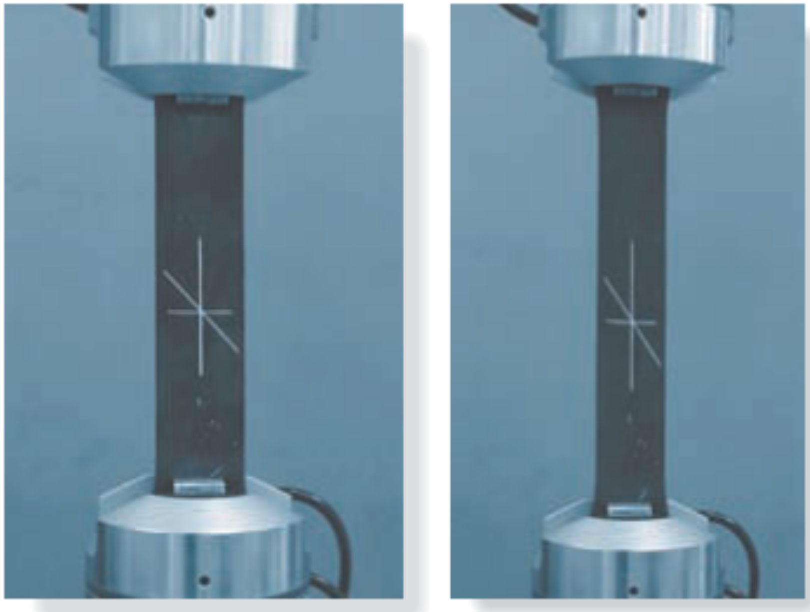
- Deformation
- Strain

Deformation

- When a force is applied to a body, it will change the body's shape and size – this is called *deformation*
- Example –
  - a rubber band undergoes a large deformation when stretched,
  - structural members have slight deformation when people are walking about on a building,
  - expansion and contraction due to changes in temperature
- Generally, the deformation of a body will not be uniform
- Hence, only consider a very short line segment and locate in the neighbourhood of a point to assume uniform deformation

Deformation

- The line segments undergo different deformation when subjected to tension
- The vertical line is lengthened
- The horizontal line is shortened
- The inclined line changes its length and rotates



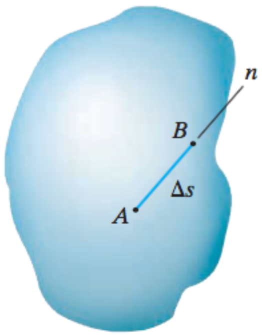
Strain

- Strain is used to describe the deformation of a body by changes in length and the chages in the angles
- It is usually obtained from measurements in experiments.

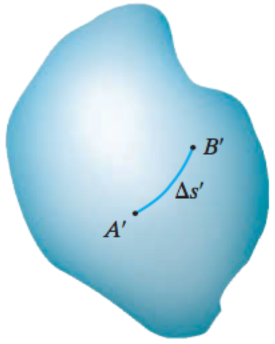
Normal Strain

- It is the change in length of a line per unit length
- Before deformation, the line AB with an original length of Δs, lying along the n axis
- After deformation, the line AB displaced to A' and B', and the line becomes a curve with a length of Δs'
- The average normal strain ε_{ave} is defined as

$$\epsilon_{avg} = \frac{\Delta s' - \Delta s}{\Delta s}$$



Undeformed body
(a)



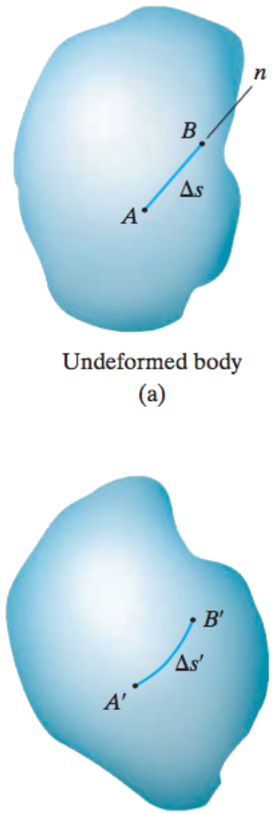
Deformed body
(b)

Normal Strain

- As point B is chosen closer and close to point A, $\Delta s \rightarrow 0$.,and hence B' will then approach A'
- Hence the limit the normal strain at point A and in the direction of n is

$$\epsilon = \lim_{B \rightarrow A \text{ along } n} \frac{\Delta s' - \Delta s}{\Delta s}$$

- Strain has no unit but typical units are
  - μm/m
  - or using percentage, i.e. 0.001 m/m = 0.1%
  - 480 μm/m is 480 μ or 480 microns



Undeformed body
(a)

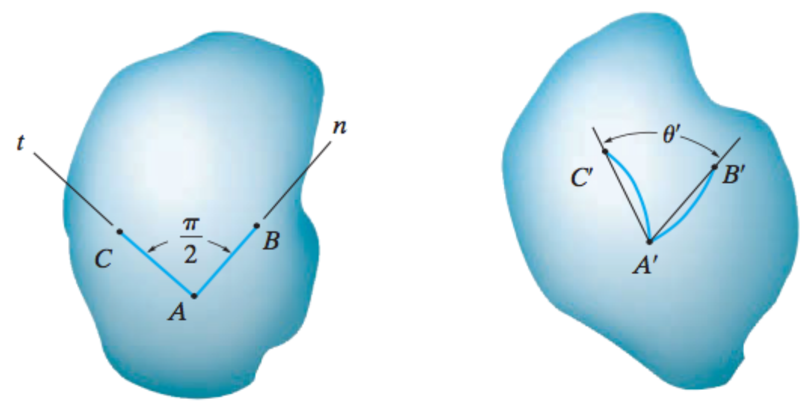
Deformed body
(b)

Shear Strain

- Deformation also causes change in direction
- Consider the lines AB and AC perpendicular to each other
- The change in angles between the line AB and AC is called Shear Strain γ
- Consider the figure below, the shear strain at point A associated with the n and t axes is

$$\gamma_{nt} = \frac{\pi}{2} - \lim_{\substack{B \to A \text{ along } n \\ C \to A \text{ along } t}} \theta'$$

- If θ' is smaller than π/2, the shear strain is positive, if greater than π/2, the shear strain is negative

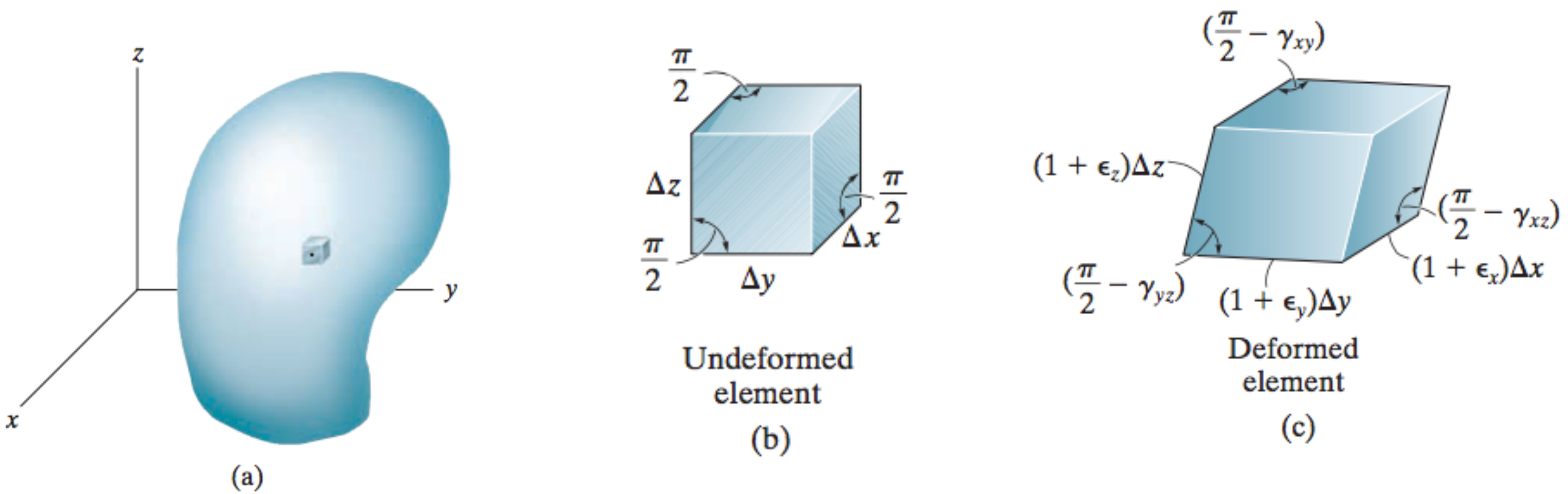


Undeformed body
(a)

Deformed body
(b)

Cartesian Strain Components

- Consider a small element in a body with undeformed dimesions Δx, Δy, and Δz
- For very small dimension, the deformed shape of a body will be a parallelepiped, since very small line segments will remain straight after the deformation

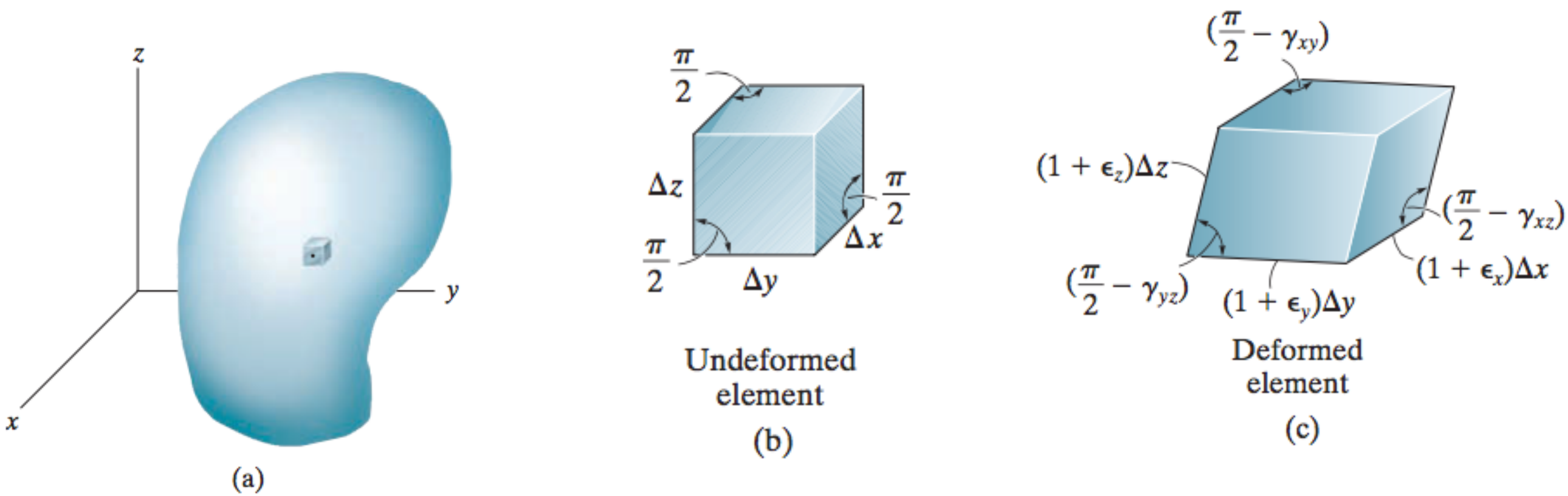


Cartesian Strain Components

- The approximate lengths of the three sides of the parallelopiped are $(1+\epsilon_x)\,\Delta x \quad (1+\epsilon_y)\,\Delta y \quad (1+\epsilon_z)\,\Delta z$
- The approximate angles between angles

$$\frac{\pi}{2}-\gamma_{xy} \qquad \frac{\pi}{2}-\gamma_{yz} \qquad \frac{\pi}{2}-\gamma_{xz}$$

- Normal strain causes a change in volume and shear strain causes a change in shape



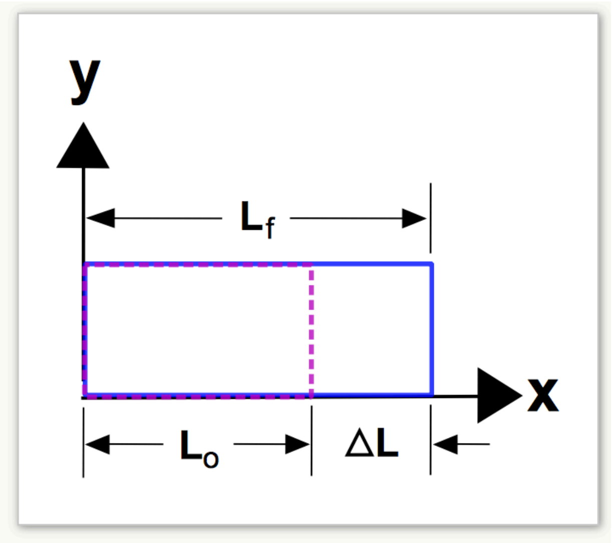
Cartesian Strain Components

- Most engineering design involves applications with only *small deformations* are allowed
- Normal strains are very small compared to 1, $\varepsilon \ll 1$ – this is called small strain analysis
- With the small strain analysis, provided θ is very small
  - $\sin\theta = \theta$
  - $\cos\theta = 1$
  - $\tan\theta = \theta$

Summary - Strain Formulation

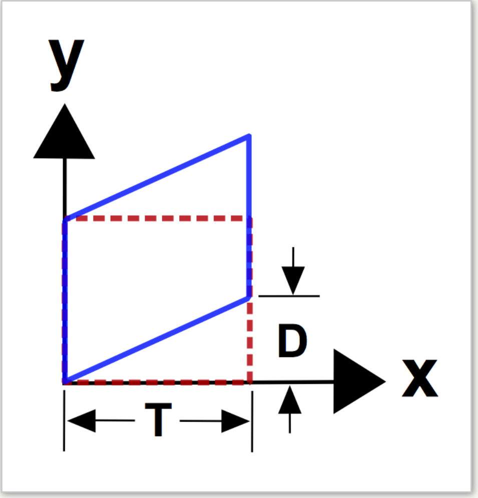
$$\epsilon_y = \frac{\partial u_y}{\partial Y}$$

$$\epsilon_x = \frac{\partial u_x}{\partial X} = \frac{L_f - L_o}{L_o} = \frac{\Delta L}{L_o}$$



Summary - Strain Formulation

$$\gamma_{xy} = \frac{\partial u_y}{\partial X} + \frac{\partial u_x}{\partial Y}$$



$$x = X$$
$$y = Y + XD/T$$

$$u_x = 0$$
$$u_y = XD/T$$

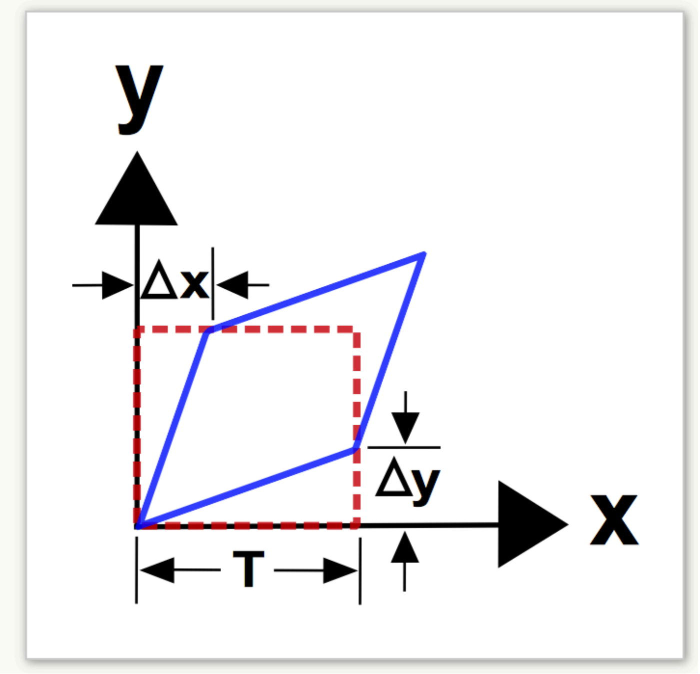
$$\gamma_{xy} = \frac{\partial u_y}{\partial X} + \frac{\partial u_x}{\partial Y} = \frac{\partial (XD/T)}{\partial X} + \frac{\partial (0)}{\partial Y} = \frac{D}{T}$$

Summary - Strain Formulation

$$\gamma_{xy} = \frac{\partial u_y}{\partial X} + \frac{\partial u_x}{\partial Y}$$

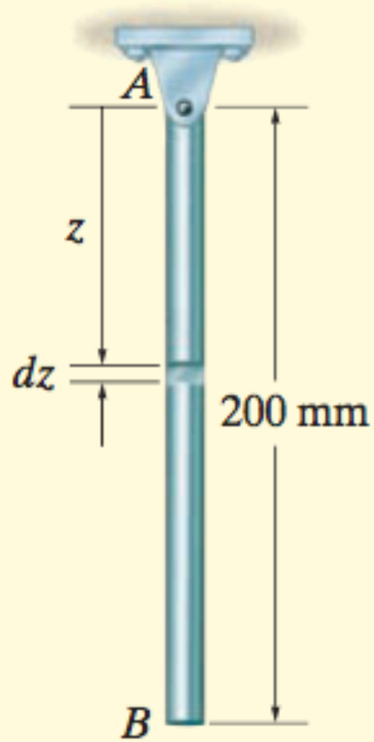
$$x = X + Y\Delta x/T$$
$$y = Y + X\Delta y/T$$

$$\gamma_{xy} = \frac{\Delta x + \Delta y}{T}$$



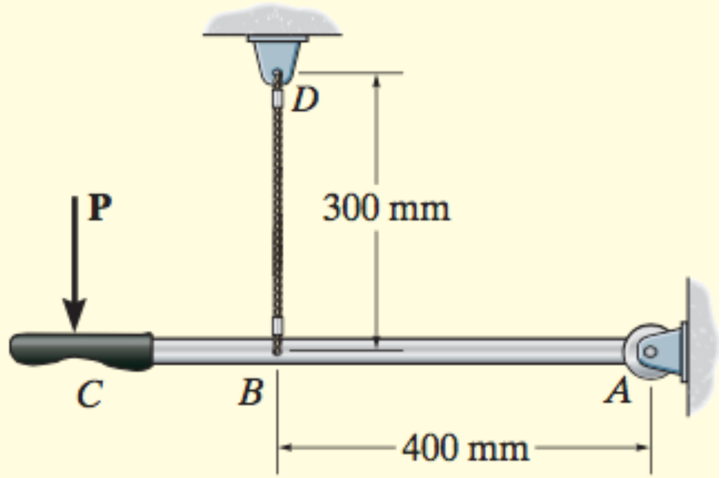
Example

The slender rod shown in Fig. 2–4 is subjected to an increase of temperature along its axis, which creates a normal strain in the rod of $\epsilon_z = 40(10^{-3})z^{1/2}$, where z is measured in meters. Determine (a) the displacement of the end B of the rod due to the temperature increase, and (b) the average normal strain in the rod.



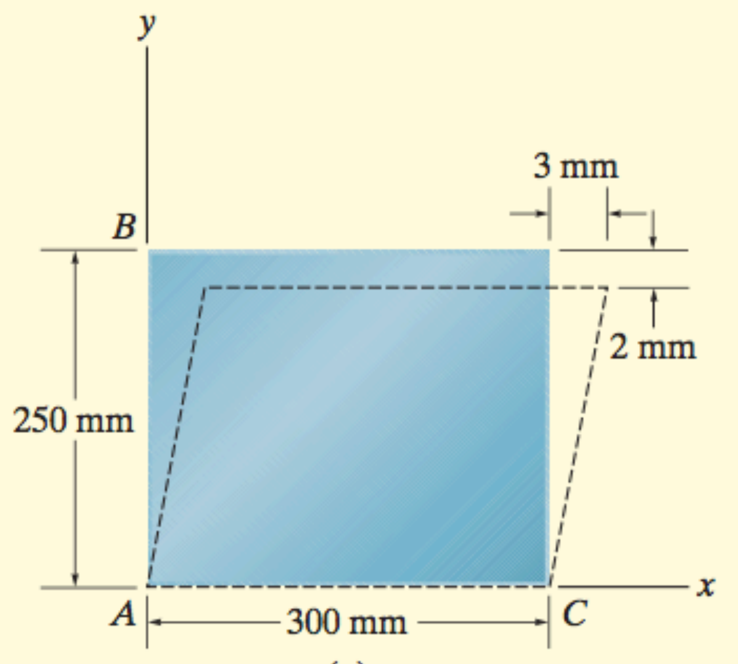
Example

When force **P** is applied to the rigid lever arm *ABC* in Fig. 2–5*a*, the arm rotates counterclockwise about pin *A* through an angle of 0.05°. Determine the normal strain developed in wire *BD*.



Example

Due to a loading, the plate is deformed into the dashed shape shown in Fig. 2–6*a*. Determine (a) the average normal strain along the side *AB*, and (b) the average shear strain in the plate at *A* relative to the *x* and *y* axes.



(a)