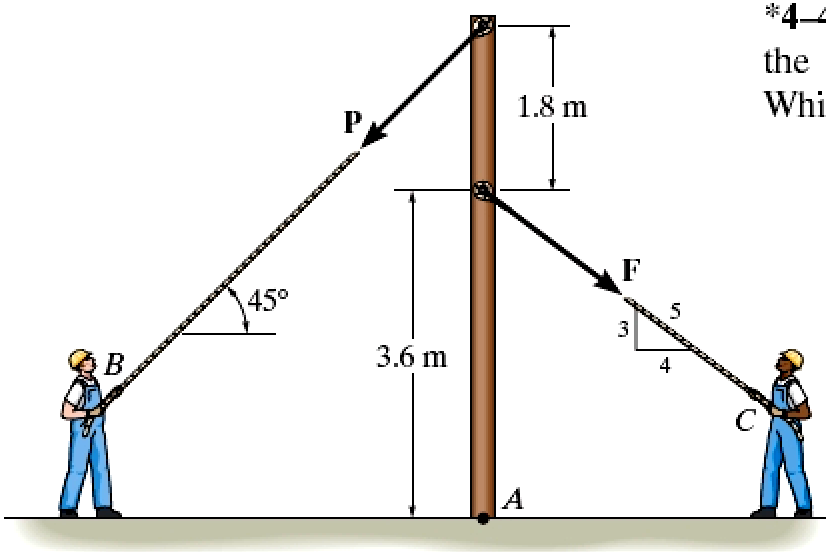


***4–4.** Two men exert forces of $F = 400$ N and $P = 250$ N on the ropes. Determine the moment of each force about A. Which way will the pole rotate, clockwise or counterclockwise?

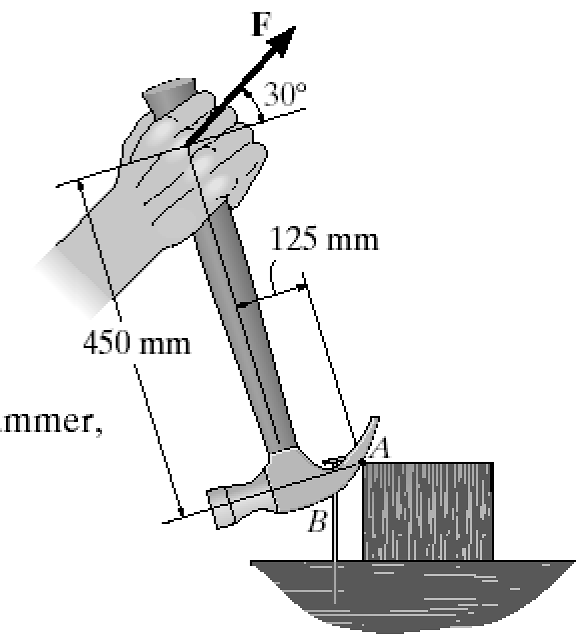
$$\circlearrowleft + \quad (M_A)_C = 400\left(\frac{4}{5}\right)(3.6) = 1152 \text{ N}\cdot\text{m} \circlearrowright \quad \textbf{Ans}$$

$$\circlearrowleft + \quad (M_A)_B = 250\,(\cos 45^\circ)\,(5.4) = 954.6 \text{ N}\cdot\text{m} \circlearrowleft \quad \textbf{Ans}$$

Since $(M_A)_C > (M_A)_B$

Clockwise **Ans**

***4–8.** The handle of the hammer is subjected to the force of $F = 1000$ N. Determine the moment of this force about the point A.

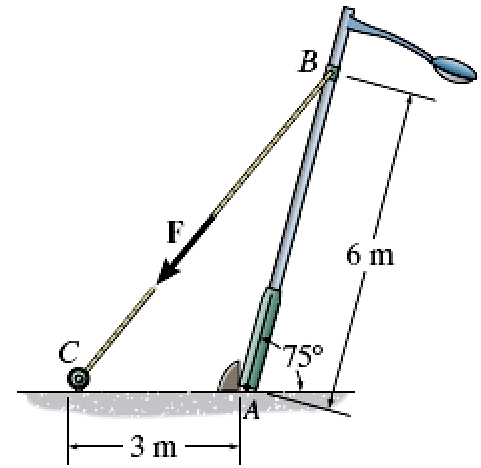


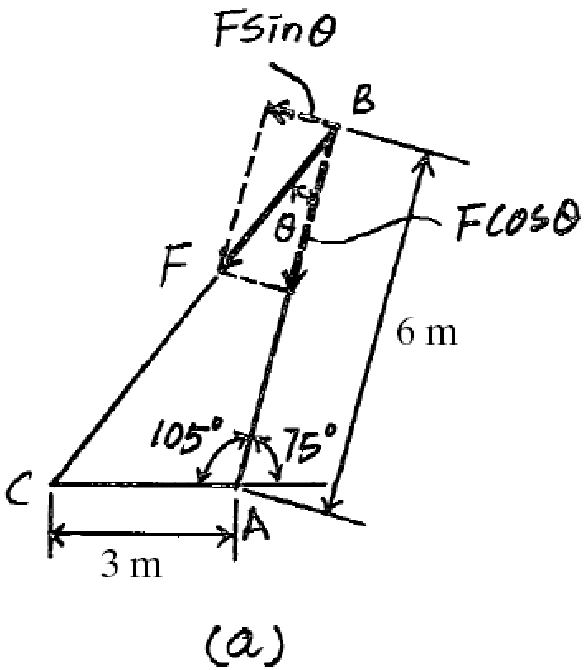
Resolving the 1000-N force into components parallel and perpendicular to the hammer, and applying the principle of moments,

$$\circlearrowleft + M_A = [-1000 \cos 30^\circ\,(450) - 1000 \sin 30^\circ\,(125)]\,(10^{-3})$$

$$= -452.2 \text{ N}\cdot\text{m} = 452 \text{ N}\cdot\text{m} \quad \text{(clockwise)} \qquad \textbf{Ans.}$$

•4–25. In order to raise the lamp post from the position shown, the force **F** on the cable must create a counterclockwise moment of 2250 N·m about point A. Determine the magnitude of **F** that must be applied to the cable.





(a)

Geometry: Applying the law of cosines to Fig. a,

$$BC^2 = 3^2 + 6^2 - 2(3)(6)\cos 105°$$

$$BC = 7.370 \text{ m}$$

Then, applying the law of sines,

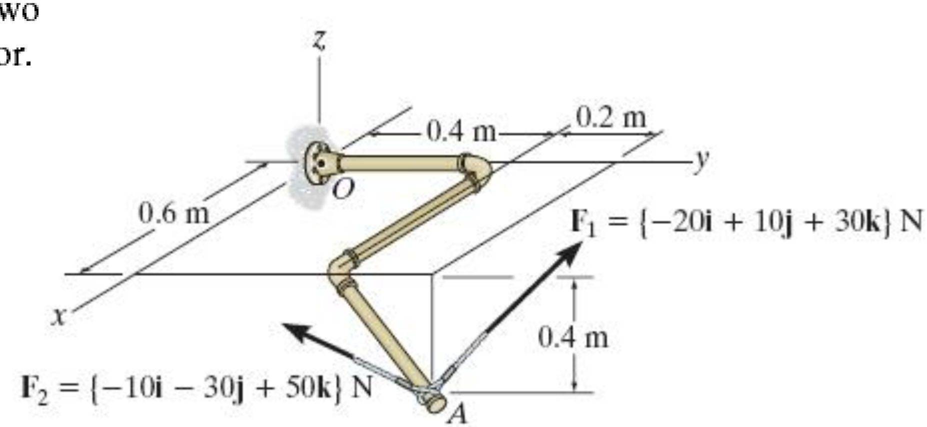
$$\frac{\sin\theta}{3} = \frac{\sin 105°}{7.37°} \qquad \theta = 23.15°$$

Moment About Point A: By resolving force **F** into components parallel and perpendicular to the lamp pole, Fig. a, and applying the principle of moments,

$$\circlearrowleft + (M_R)_A = \Sigma Fd; \quad 2250 = F\sin 23.15°\,(6)$$

$$F = 953.9 \text{ N} \qquad \textbf{Ans.}$$

4–39. Determine the resultant moment produced by the two forces about point O. Express the result as a Cartesian vector.



A (0.6, 0.6, -0.4)

Position Vector: The position vector $\mathbf{r}_{OA}$, Fig. *a*, must be determined first.

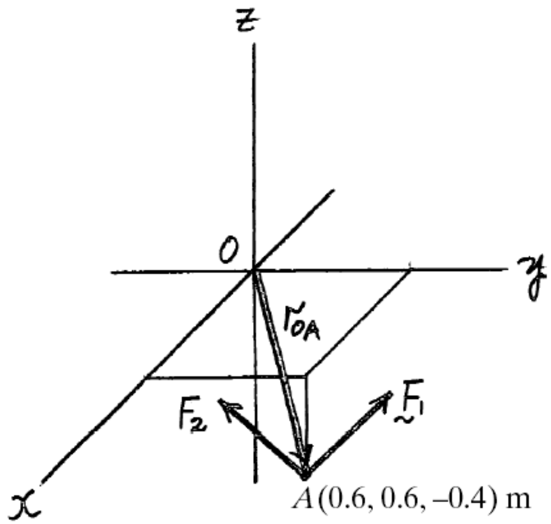
$$\mathbf{r}_{OA} = (0.6 - 0)\mathbf{i} + (0.6 - 0)\mathbf{j} + (-0.4 - 0)\mathbf{k} = [0.6\mathbf{i} + 0.6\mathbf{j} - 0.4\mathbf{k}]\text{ m}$$

Resultant Moment: The resultant moment of $\mathbf{F}_1$ and $\mathbf{F}_2$ about point O can be determined by

$$(\mathbf{M}_R)_O = \mathbf{r}_{OA} \times \mathbf{F}_1 + \mathbf{r}_{OA} \times \mathbf{F}_2$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6 & 0.6 & -0.4 \\ -20 & 10 & 30 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6 & 0.6 & -0.4 \\ -10 & -30 & 50 \end{vmatrix}$$

$$= [40\mathbf{i} - 36\mathbf{j} + 6\mathbf{k}]\text{ N}\cdot\text{m}$$ **Ans.**



(a)

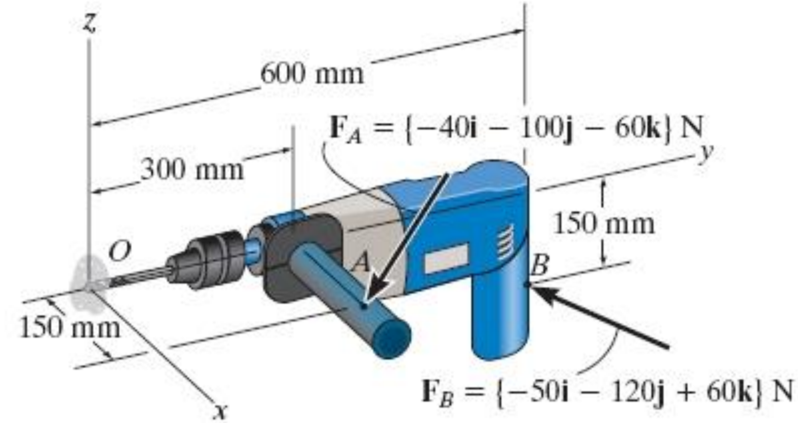
Or we can apply the principle of moments which gives

$$(\mathbf{M}_R)_O = \mathbf{r}_{OA} \times (\mathbf{F}_1 + \mathbf{F}_2)$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.6 & 0.6 & -0.4 \\ -30 & -20 & 80 \end{vmatrix}$$

$$= [40\mathbf{i} - 36\mathbf{j} + 6\mathbf{k}]\text{ N}\cdot\text{m}$$ **Ans.**

4–43. Determine the moment produced by each force about point O located on the drill bit. Express the results as Cartesian vectors.



Position Vector: The position vectors $\mathbf{r}_{OA}$ and $\mathbf{r}_{OB}$, Fig. *a*, must be determined first.

$$\mathbf{r}_{OA} = (0.15-0)\mathbf{i}+(0.3-0)\mathbf{j}+(0-0)\mathbf{k} = [0.15\mathbf{i}+0.3\mathbf{j}]\,\text{m}$$

$$\mathbf{r}_{OB} = (0-0)\mathbf{i}+(0.6-0)\mathbf{j}+(-0.15-0)\mathbf{k} = [0.6\mathbf{j}-0.15\mathbf{k}]\,\text{m}$$

Vector Cross Product: The moment of $\mathbf{F}_A$ about point O is

$$(\mathbf{M}_R)_O = \mathbf{r}_{OA} \times \mathbf{F}_A = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.15 & 0.3 & 0 \\ -40 & -100 & -60 \end{vmatrix} = [-18\mathbf{i}+9\mathbf{j}-3\mathbf{k}]\,\text{N}\cdot\text{m}$$

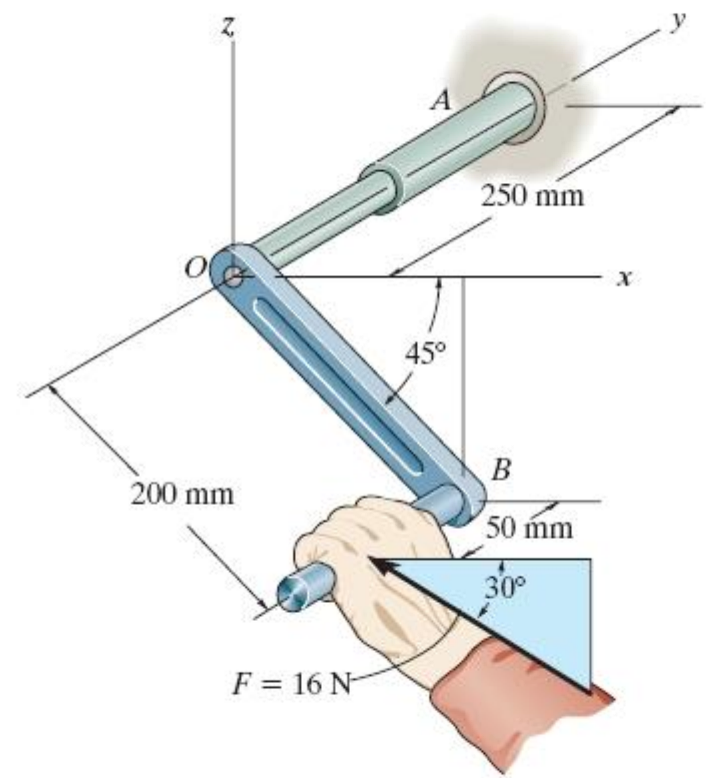
Ans.

The moment of $\mathbf{F}_B$ about point O is

$$(\mathbf{M}_R)_O = \mathbf{r}_{OB} \times \mathbf{F}_B = \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0 & 0.6 & -0.15 \\ -50 & -120 & 60 \end{vmatrix} = [18\mathbf{i}+7.5\mathbf{j}+30\mathbf{k}]\,\text{N}\cdot\text{m}$$

Ans.

•**4–57.** Determine the magnitude of the moment that the force **F** exerts about the *y* axis of the shaft. Solve the problem using a Cartesian vector approach and using a scalar approach.



a) ***Vector Analysis***

Position Vector and Force Vector :

$$\mathbf{r}_{OB} = \{0.2\cos 45°\mathbf{i} - 0.2\sin 45°\mathbf{k}\}\ \text{m} = \{0.1414\mathbf{i} - 0.1414\mathbf{k}\}\ \text{m}$$

$$\mathbf{F} = 16\{-\cos 30°\mathbf{i} + \sin 30°\mathbf{k}\}\ \text{N} = \{-13.856\mathbf{i} + 8.00\mathbf{k}\}\ \text{N}$$

Moment of Force F About y Axis : The unit vector along the y axis is j.

$$M_y = \mathbf{j}\cdot(\mathbf{r}_{OB} \times \mathbf{F})$$

$$= \begin{vmatrix} 0 & 1 & 0 \\ 0.1414 & 0 & -0.1414 \\ -13.856 & 0 & 8 \end{vmatrix}$$

$$= 0 - 1[0.1414(8) - (-13.856)(-0.1414)] + 0$$

$$= 0.828\ \text{N}\cdot\text{m}$$ **Ans**

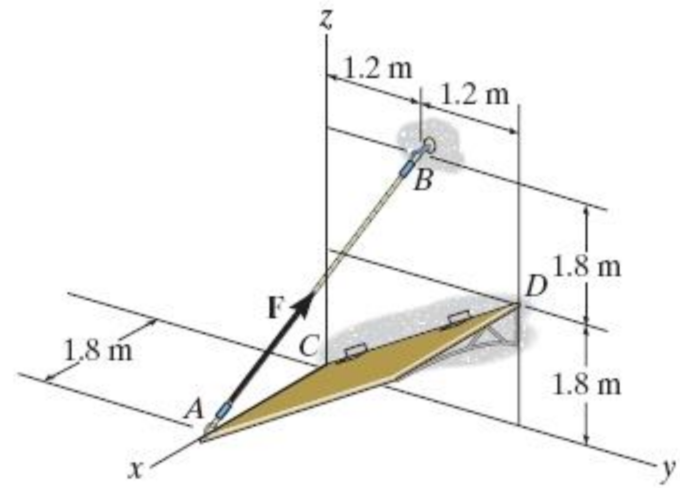
B (0.2 cos45, 0, -0.2 sin45)

b) ***Scalar Analysis***

$$M_y = \Sigma M_y; \quad M_y = 16\cos 30°(0.2\sin 45°) - 16\sin 30°(0.2\cos 45°)$$

$$= 0.828\ \text{N}\cdot\text{m}$$ **Ans**

•4–61. If the tension in the cable is $F = 700$ N, determine the magnitude of the moment produced by this force about the hinged axis, CD, of the panel.



$A(1.8, 0, 0)$ m $\quad B(0, 1.2, 3.6)$ m $\quad D(0, 2.4, 1.8)$ m

Moment About the CD axis: Either position vector $\mathbf{r}_{CA}$ or $\mathbf{r}_{DB}$, Fig. a, can be used to determine the moment of **F** about the CD axis.

$$\mathbf{r}_{CA} = (1.8-0)\mathbf{i} + (0-0)\mathbf{j} + (0-0)\mathbf{k} = [1.8\mathbf{i}] \text{ m}$$

$$\mathbf{r}_{DB} = (0-0)\mathbf{i} + (1.2-2.4)\mathbf{j} + (3.6-1.8)\mathbf{k} = [-1.2\mathbf{j} + 1.8\mathbf{k}] \text{ m}$$

$$\mathbf{F} = F\mathbf{u}_{AB} = 700\left[\frac{(0-1.8)\mathbf{i} + (1.2-0)\mathbf{j} + (3.6-0)\mathbf{k}}{\sqrt{(0-1.8)^2 + (1.2-0)^2 + (3.6-0)^2}}\right] = [-300\mathbf{i} + 200\mathbf{j} + 600\mathbf{k}] \text{ N}$$

The unit vector $\mathbf{u}_{CD}$, Fig. a, that specifies the direction of the CD axis is given by

$$\mathbf{u}_{CD} = \left[\frac{(0-0)\mathbf{i} + (2.4-0)\mathbf{j} + (1.8-0)\mathbf{k}}{\sqrt{(0-0)^2 + (2.4-0)^2 + (1.8-0)^2}}\right] = \frac{4}{5}\mathbf{j} + \frac{3}{5}\mathbf{k}$$

Thus, the magnitude of the moment of **F** about the CD axis is given by

$$M_{CD} = \mathbf{u}_{CD} \cdot \mathbf{r}_{CA} \times \mathbf{F} = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 1.8 & 0 & 0 \\ -300 & 200 & 600 \end{vmatrix}$$

$$= 0 - \frac{4}{5}[1.8(600) - (-300)(0)] + \frac{3}{5}[1.8(200) - (-300)(0)]$$

$$= -648 \text{ N} \cdot \text{m}$$ **Ans.**

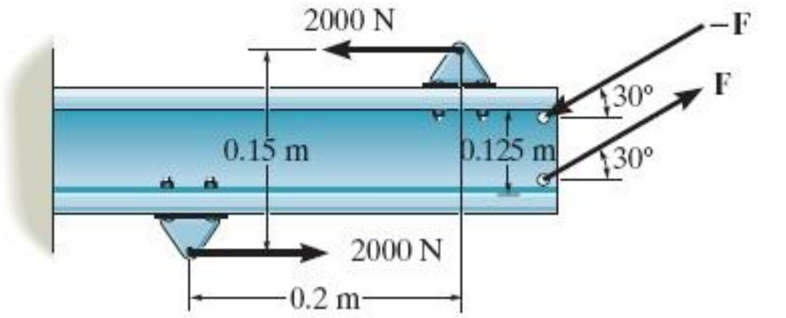
or

$$M_{CD} = \mathbf{u}_{CD} \cdot \mathbf{r}_{DB} \times \mathbf{F} = \begin{vmatrix} 0 & \frac{4}{5} & \frac{3}{5} \\ 0 & -1.2 & 1.8 \\ -300 & 200 & 600 \end{vmatrix}$$

$$= -648 \text{ N} \cdot \text{m}$$

The negative sign indicates that $\mathbf{M}_{CD}$ acts in the opposite sense to that of $\mathbf{u}_{CD}$.

***4–80.** Two couples act on the beam. Determine the magnitude of **F** so that the resultant couple moment is 450 N · m, counterclockwise. Where on the beam does the resultant couple moment act?

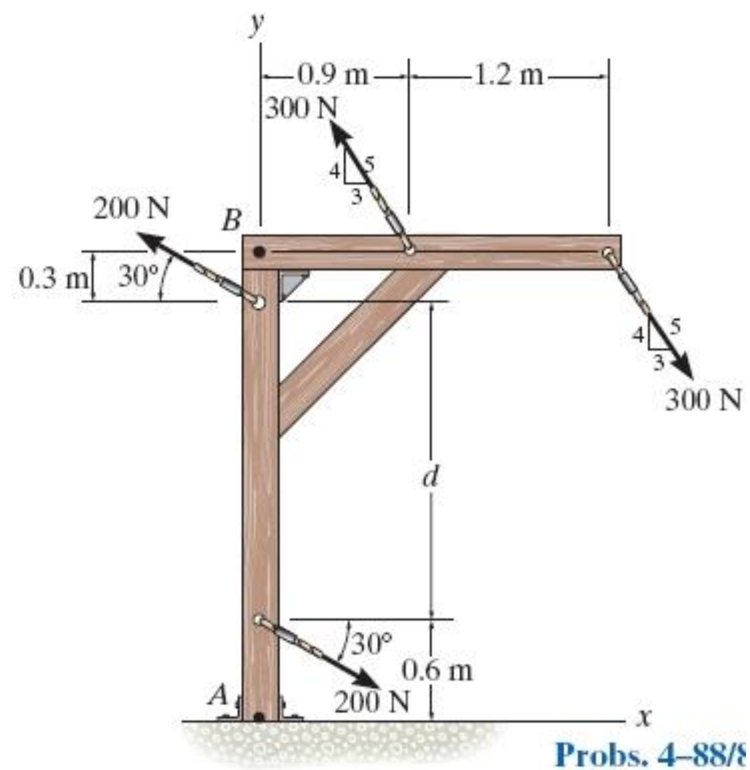


$$\curvearrowleft + M_R = \Sigma M; \qquad 450 = 2000\,(0.15) + F\cos 30^\circ\,(0.125)$$

$$F = 1386 \text{ N} \qquad \textbf{Ans}$$

The resultant couple moment is a free vector. It can act at any point on the beam.

•4–89. Two couples act on the frame. If $d = 1.2$ m, determine the resultant couple moment. Compute the result by resolving each force into x and y components and (a) finding the moment of each couple and (b) summing the moments of all the force components about point A.



Probs. 4–88/8

(a) $\curvearrowleft + M_C = 200\cos 30^\circ\,(1.2) - 300\left(\dfrac{4}{5}\right)(1.2)$

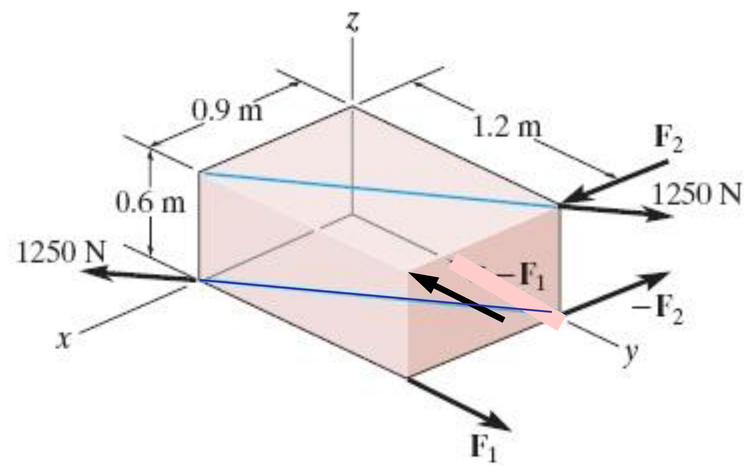
$= -80.15 \text{ N}\cdot\text{m} = 80.15 \text{ N}\cdot\text{m} \curvearrowright$ **Ans**

(b) $\curvearrowleft + M_C = -200\cos 30^\circ\,(0.6) + 200\cos 30^\circ\,(1.8) + 300\left(\dfrac{4}{5}\right)(0.9) +$

$300\left(\dfrac{3}{5}\right)(2.1) - 300\left(\dfrac{4}{5}\right)(2.1) - 300\left(\dfrac{3}{5}\right)(2.1)$

$= -80.15 \text{ N}\cdot\text{m} = 80.15 \text{ N}\cdot\text{m} \curvearrowright$ **Ans**

4–102. If $F_1 = 500$ N and $F_2 = 1000$ N, determine the magnitude and coordinate direction angles of the resultant couple moment.



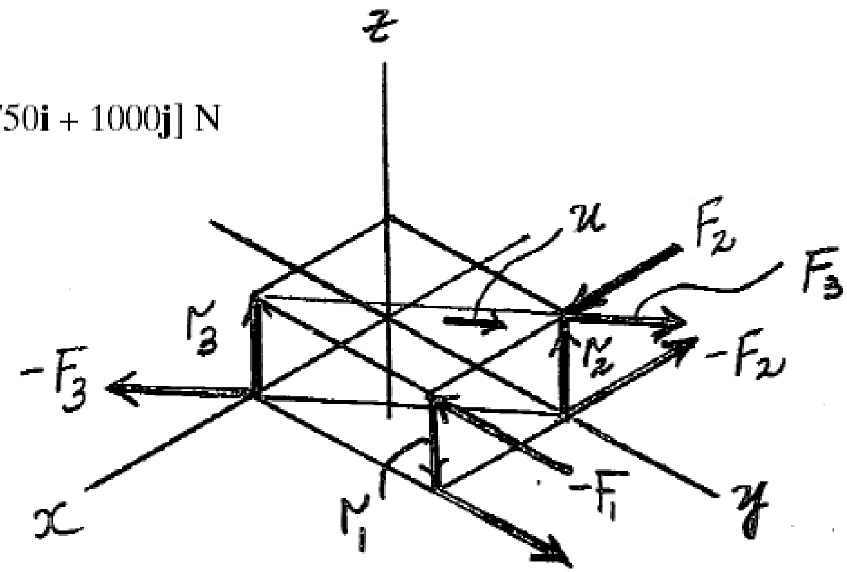
Couple Moment: The position vectors $\mathbf{r}_1$, $\mathbf{r}_2$, and $\mathbf{r}_3$, Fig. *a*, must be determined first.

$\mathbf{r}_1 = [-0.6\mathbf{k}]$ m $\qquad \mathbf{r}_2 = [0.6\mathbf{k}]$ m $\qquad \mathbf{r}_3 = [0.6\mathbf{k}]$ m

The force vectors $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ are given by

$\mathbf{F}_1 = [500\mathbf{j}]$ N $\qquad \mathbf{F}_2 = [1000\mathbf{i}]$ N

$$\mathbf{F}_3 = F_3\mathbf{u} = 1250\left[\frac{(0-0.9)\mathbf{i} + (1.2-0)\mathbf{j} + (0.6-0.6)\mathbf{k}}{\sqrt{(0-0.9)^2 + (1.2-0)^2 + (0.6-0.6)^2}}\right] = [-750\mathbf{i} + 1000\mathbf{j}]\text{ N}$$



(a)

Thus,

$$\mathbf{M}_1 = \mathbf{r}_1 \times \mathbf{F}_1 = (-0.6\mathbf{k}) \times (500\mathbf{j}) = [300\mathbf{i}] \text{ N}\cdot\text{m}$$

$$\mathbf{M}_2 = \mathbf{r}_2 \times \mathbf{F}_2 = (0.6\mathbf{k}) \times (1000\mathbf{i}) = [600\mathbf{j}] \text{ N}\cdot\text{m}$$

$$\mathbf{M}_3 = \mathbf{r}_3 \times \mathbf{F}_3 = (0.6\mathbf{k}) \times (-750\mathbf{i} + 1000\mathbf{j}) = [-600\mathbf{i} - 450\mathbf{j}] \text{ N}\cdot\text{m}$$

Resultant Moment: The resultant couple moment is given by

$$(\mathbf{M}_C)_R = \Sigma\mathbf{M}_C; \qquad (\mathbf{M}_C)_R = \mathbf{M}_1 + \mathbf{M}_2 + \mathbf{M}_3$$
$$= (300\mathbf{i}) + (600\mathbf{j}) + (-600\mathbf{i} - 450\mathbf{j})$$
$$= [-300\mathbf{i} + 150\mathbf{j}] \text{ N}\cdot\text{m}$$

The magnitude of the couple moment is

$$(\mathbf{M}_C)_R = \sqrt{[(M_C)_R]_x^{\,2} + [(M_C)_R]_y^{\,2} + [(M_C)_R]_z^{\,2}}$$
$$= \sqrt{(-300)^2 + (150)^2 + (0)^2}$$
$$= 335.41 \text{ N}\cdot\text{m} = 335 \text{ N}\cdot\text{m}$$

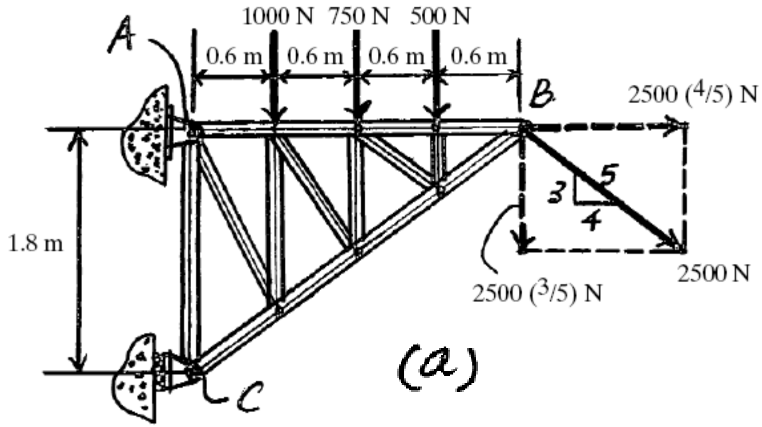
The coordinate angles of $(\mathbf{M}_C)_R$ are

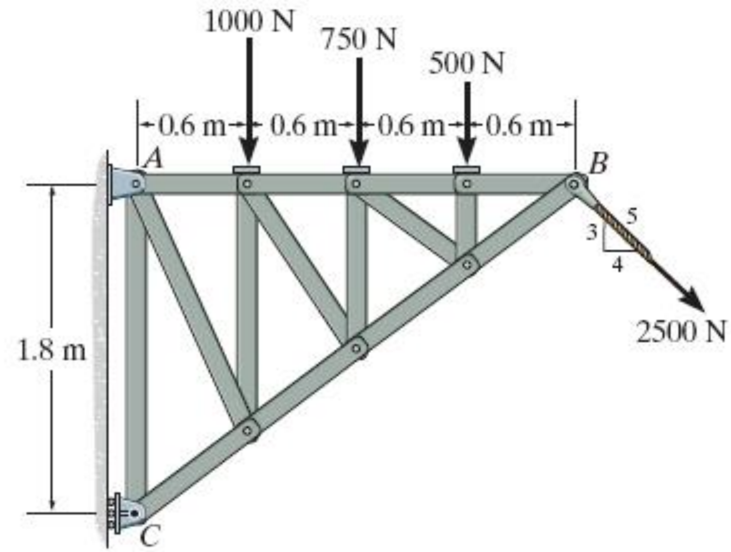
$$\alpha = \cos^{-1}\left(\frac{[(M_C)R]_x}{(M_C)_R}\right) = \cos^{-1}\left(\frac{-300}{335.41}\right) = 153.4°$$

$$\beta = \cos^{-1}\left(\frac{[(M_C)R]_y}{(M_C)_R}\right) = \cos^{-1}\left(\frac{150}{335.41}\right) = 63.4°$$

$$\gamma = \cos^{-1}\left(\frac{[(M_C)R]_z}{(M_C)_R}\right) = \cos^{-1}\left(\frac{0}{335.41}\right) = 90°$$

*4–104. Replace the force system acting on the truss by a resultant force and couple moment at point C.





Equivalent Resultant Force: The 2500-N force is resolved into its x and y components, Fig. a. Summing these force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow}\Sigma(F_R)_x = \Sigma F_x; \qquad (F_R)_x = 2500\left(\frac{4}{5}\right) = 2000\text{ N} \rightarrow$$

$$+\uparrow\Sigma(F_R)_y = \Sigma F_y; \qquad (F_R)_y = -1000 - 750 - 500 - 2500\left(\frac{3}{5}\right) = -3750\text{ N} = 3750\text{ N}\downarrow$$

The magnitude of the resultant force $\mathbf{F}_R$ is given by

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{2000^2 + 3750^2} = 4250\text{ N} \quad \textbf{Ans.}$$

The angle θ of $\mathbf{F}_R$ is

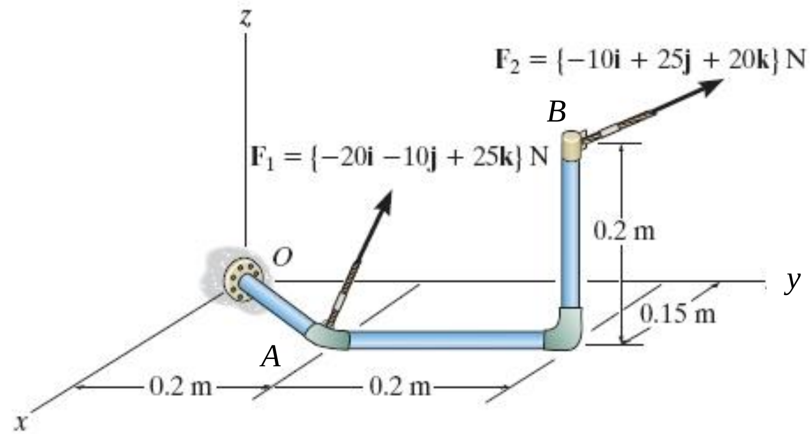
$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left[\frac{3750}{2000}\right] = 61.93° = 61.9° \quad \textbf{Ans.}$$

Equivalent Couple Moment: Summing the moment about point C, $\curvearrowleft + (M_R)_C = \Sigma M_C$;

$$(M_R)_C = -1000\,(0.6) - 750\,(1.2) - 500\,(1.8) - 2500\left(\frac{3}{5}\right)(2.4) - 2500\left(\frac{4}{5}\right)(1.8)$$

$$= -9600\text{ N}\cdot\text{m} = 9.60\text{ kN}\cdot\text{m (clockwise)} \quad \textbf{Ans.}$$

***4–116.** Replace the force system acting on the pipe assembly by a resultant force and couple moment at point O. Express the results in Cartesian vector form.



A (0.15, 0.2, 0)
B (0.15, 0.4, 0.2)

Equivalent Resultant Force: The resultant force $\mathbf{F}_R$ can be determined from

$$\mathbf{F}_R = \Sigma\mathbf{F}; \qquad \mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2$$

$$= (-20\mathbf{i} - 10\mathbf{j} + 25\mathbf{k}) + (-10\mathbf{i} + 25\mathbf{j} + 20\mathbf{k})$$

$$= [-30\mathbf{i} + 15\mathbf{j} + 45\mathbf{k}] \text{ N}$$ **Ans.**

Equivalent Resultant Couple Moment: The position vectors $\mathbf{r}_{OA}$ and $\mathbf{r}_{OB}$, are

$$\mathbf{r}_{OA} = (0.15 - 0)\mathbf{i} + (0.2 - 0)\mathbf{j} + (0 - 0)\mathbf{k} = [0.15\mathbf{i} + 0.2\mathbf{j}] \text{ m}$$

$$\mathbf{r}_{OB} = (0.15 - 0)\mathbf{i} + (0.4 - 0)\mathbf{j} + (0.2 - 0)\mathbf{k} = [0.15\mathbf{i} + 0.4\mathbf{j} + 0.2\mathbf{k}] \text{ m}$$

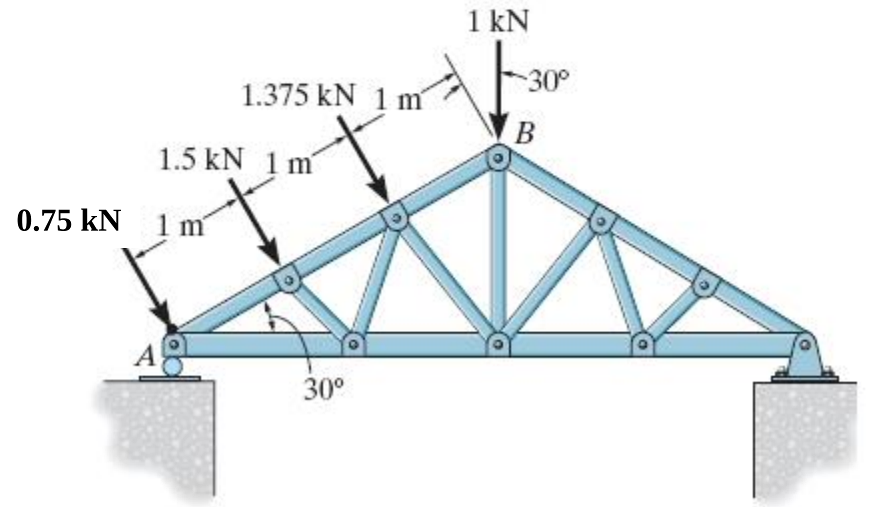
Thus, the resultant couple moment about point O is

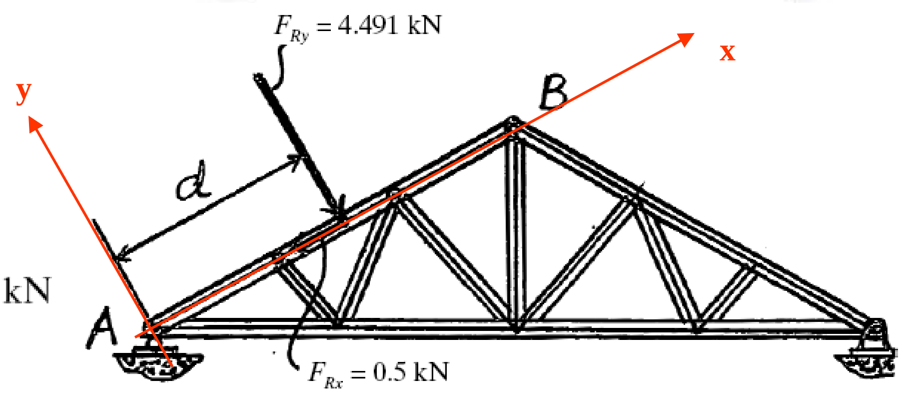
$$M_W = \Sigma\mathbf{F}_O; \qquad (\mathbf{M}_R)_O = \mathbf{r}_{OA} \times \mathbf{F}_1 + \mathbf{r}_{OB} \times \mathbf{F}_2$$

$$= \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.15 & 0.4 & 0.2 \\ -10 & 25 & 20 \end{vmatrix} + \begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ 0.15 & 0.4 & 0.2 \\ -10 & 25 & 20 \end{vmatrix}$$

$$= [8\mathbf{i} - 8.75\mathbf{j} + 10.25\mathbf{k}] \text{ N} \cdot \text{m}$$ **Ans.**

•**4–121.** The system of four forces acts on the roof truss. Determine the equivalent resultant force and specify its location along AB, measured from point A.



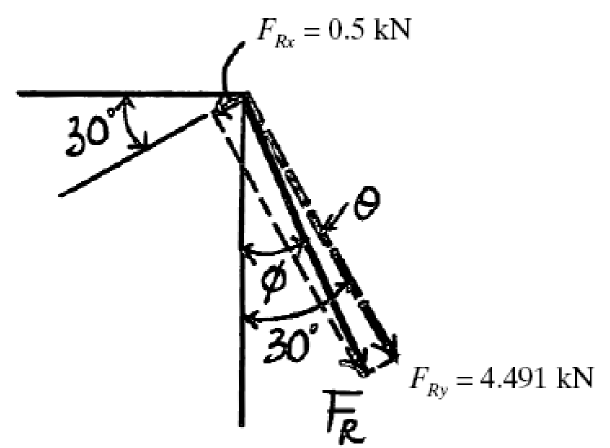


$\swarrow + F_{Rx} = \Sigma F_x;$ $\quad F_{Rx} = 1 \sin 30° = 0.5$ kN

$\searrow + F_{Ry} = \Sigma F_y;$ $\quad F_{Ry} = 0.75 + 1.5 + 1.375 + 1 \cos 30° = 4.491$ kN

$F_R = \sqrt{(0.5)^2 + (4.491)^2} = 4.52$ kN **Ans**

$\theta = \tan^{-1}\left(\frac{0.5}{4.491}\right) = 6.35°$

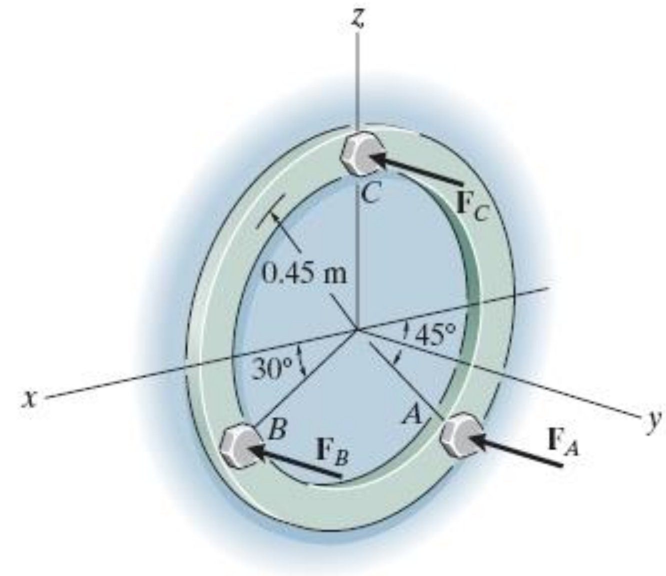


$\phi = 30° - 6.35° = 23.6°$ **Ans**

$\circlearrowleft + M_{RA} = \Sigma M_A;$ $\quad 4.491\,(d) = 1\,(1.5) + 2\,(1.375) + 3 \cos 30°\,(1)$

$d = 1.52$ m **Ans**

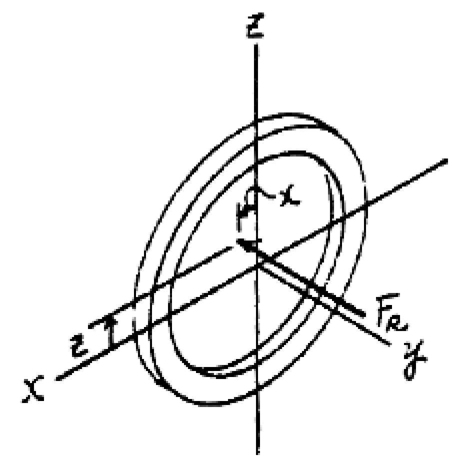
*4–132. Three parallel bolting forces act on the circular plate. Determine the resultant force, and specify its location (x, z) on the plate. $F_A = 1000$ N, $F_B = 500$ N, and $F_C = 2000$ N.



Equivalent Force :

$F_R = \Sigma F_y;$ $-F_R = -2000 - 1000 - 500$

$F_R = 3500\text{ N}$ **Ans**



Location of Resultant Foce :

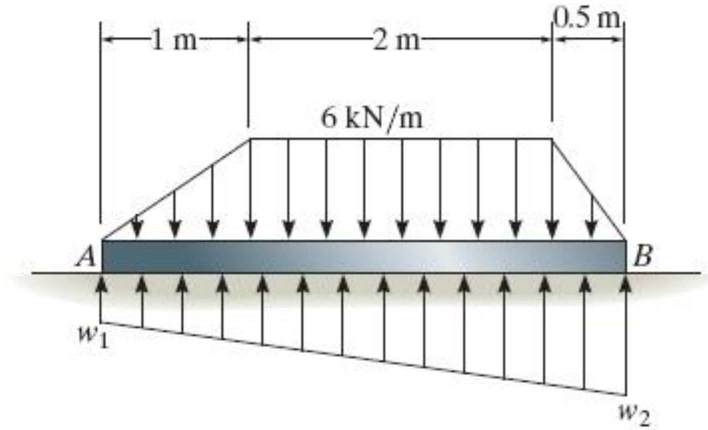
$M_{R_x} = \Sigma M_x;$ $3500\,(z) = 2000\,(0.45) - 1000\,(0.45\sin 45°) - 500\,(0.45\sin 30°)$

$z = 0.134\text{ m}$ **Ans**

$M_{R_z} = \Sigma M_z;$ $-3500\,(x) = 1000\,(0.45\cos 45°) - 500\,(0.45\cos 30°)$

$x = -0.0352\text{ m}$ **Ans**

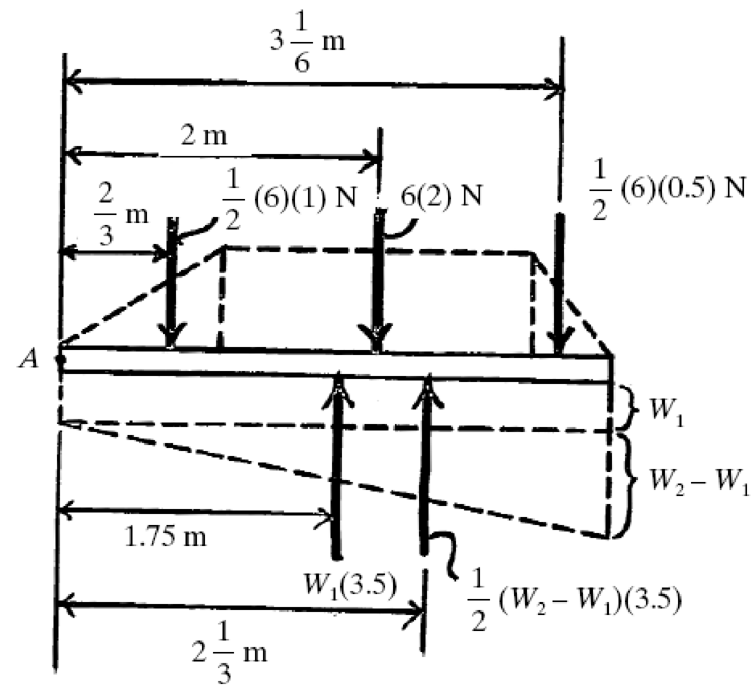
4–147. Determine the intensities w_1 and w_2 of the distributed loading acting on the bottom of the slab so that this loading has an equivalent resultant force that is equal but opposite to the resultant of the distributed loading acting on the top of the plate.



$+\uparrow F_R = \Sigma F;$

$$0 = w_1(3.5) + \frac{1}{2}(w_2 - w_1)(3.5) - \frac{1}{2}(6)(1) - 6(2) - \frac{1}{2}(6)(0.5)$$

$$w_1 + w_2 = 9.42857 \qquad (1)$$



$\circlearrowleft + M_{RA} = \Sigma M_A;$

$$0 = w_1(3.5)(1.75) + \frac{1}{2}(w_2 - w_1)(3.5)\left(2\frac{1}{3}\right) - \frac{1}{2}(6)(1)\left(\frac{2}{3}\right) - 6(2)(2) - \frac{1}{2}(6)(0.5)\left(3\frac{1}{6}\right)$$

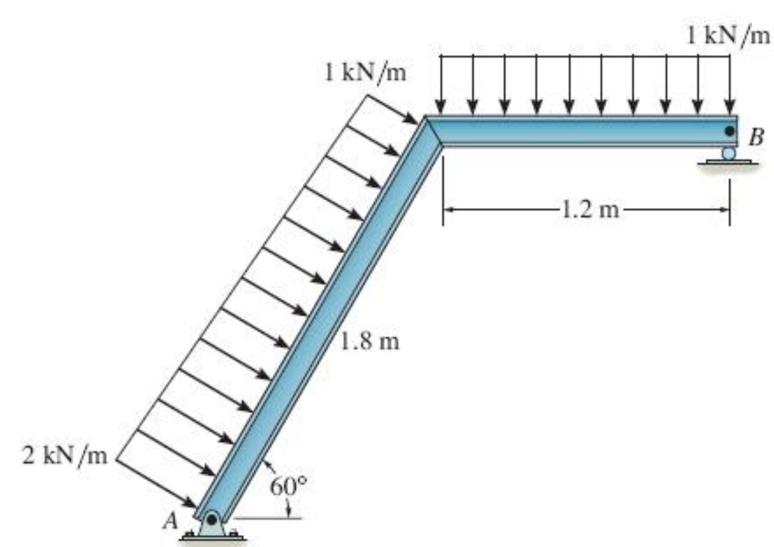
$$2.042\,w_1 + 4.083 w_2 = 30.75 \qquad (2)$$

Solving Eqs. (1) and (2),

$w_1 = 3.766$ kN/m **Ans**

$w_2 = 5.633$ kN/m **Ans**

4–155. Replace the loading by an equivalent resultant force and couple moment at point *A*.



$$F_1 = \frac{1}{2}\ (1.8)\ (1) = 0.9\ \text{kN}$$

$$F_2 = (1.8)\ (1) = 1.8\ \text{kN}$$

$$F_3 = (1.2)\ (1) = 1.2\ \text{kN}$$

$\overset{+}{\rightarrow} F_{Rx} = \Sigma F_x;\qquad F_{Rx} = 0.9 \sin 60° + 1.8 \sin 60° = 2.338\ \text{kN}$

$+\downarrow F_{Ry} = \Sigma F_y;\qquad F_{Ry} = 0.9 \cos 60° + 1.8 \cos 60° + 1.2 = 2.55\ \text{kN}$

$F_R = \sqrt{(2.338)^2 + (2.55)^2} = 3.460\ \text{kN}$ **Ans**

$\theta = \tan^{-1}\left(\frac{2.55}{2.338}\right) = 47.5°$ **Ans**

$\circlearrowleft + M_{RA} = \Sigma M_A;\qquad M_{RA} = 0.9\ (0.6) + 1.8\ (0.9) + 1.2\ (1.8 \cos 60° + 0.6)$

$= 3.96\ \text{kN}\cdot\text{m}$ $\circlearrowright$ **Ans**