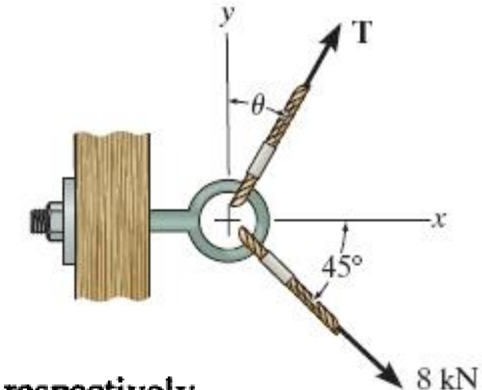
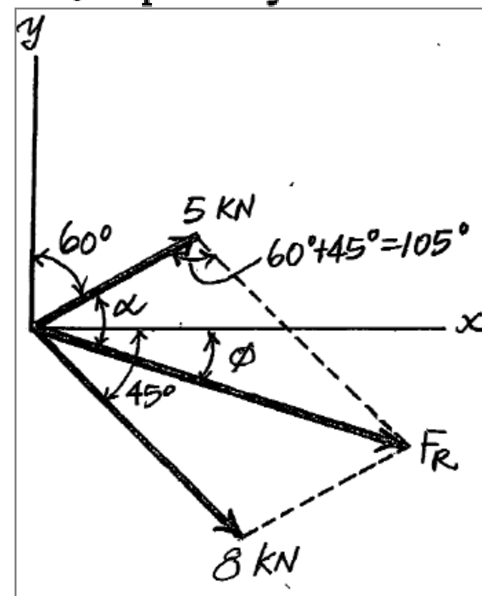
2–2. If $\theta = 60^\circ$ and $T = 5$ kN, determine the magnitude of the resultant force acting on the eyebolt and its direction measured clockwise from the positive x axis.



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.



(a)

Applying the law of cosines to Fig. *b*,

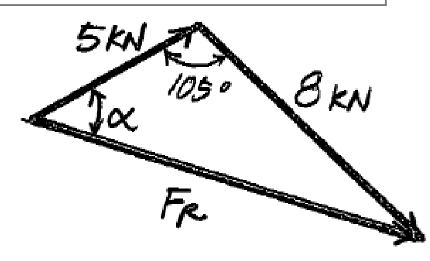
$$F_R = \sqrt{5^2 + 8^2 - 2(5)(8)\cos 105^\circ}$$
$$= 10.47 \text{ kN} = 10.5 \text{ kN} \qquad \textbf{Ans.}$$

Applying the law of sines to Fig. *b* and using this result, yields

$$\frac{\sin\alpha}{8} = \frac{\sin 105^\circ}{10.47} \qquad \alpha = 47.54^\circ$$

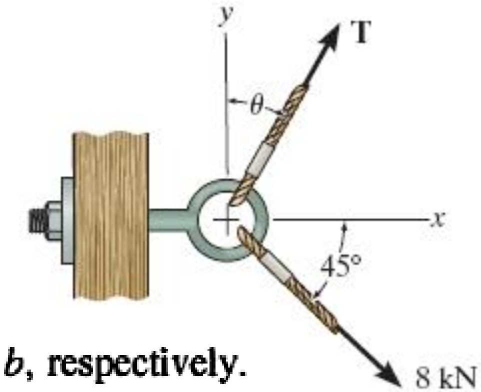
Thus, the direction angle ϕ of $\mathbf{F}_R$ measured clockwise from the positive x axis is

$$\phi = \alpha - 30^\circ = 47.54^\circ - 30^\circ = 17.5^\circ \qquad \textbf{Ans.}$$



(b)

2–3. If the magnitude of the resultant force is to be 9 kN directed along the positive x axis, determine the magnitude of force **T** acting on the eyebolt and its angle θ.



The parallelogram law of addition and the triangular rule are shown in Figs. *a* and *b*, respectively.

Applying the law of cosines to Fig. *b*,

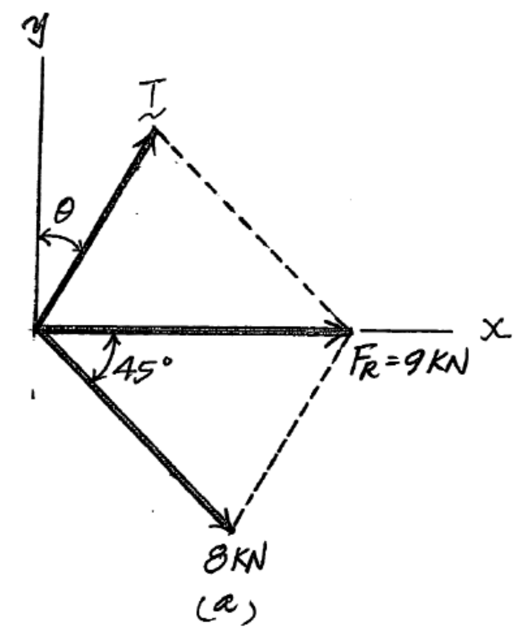
$$T = \sqrt{8^2 + 9^2 - 2(8)(9)\cos 45^\circ}$$

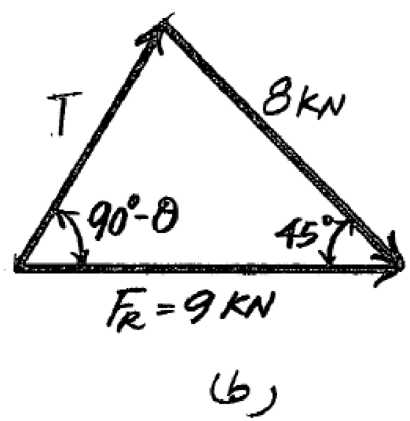
$$= 6.571 \text{ kN} = 6.57 \text{ kN} \qquad \textbf{Ans.}$$

Applying the law of sines to Fig. *b* and using this result, yields

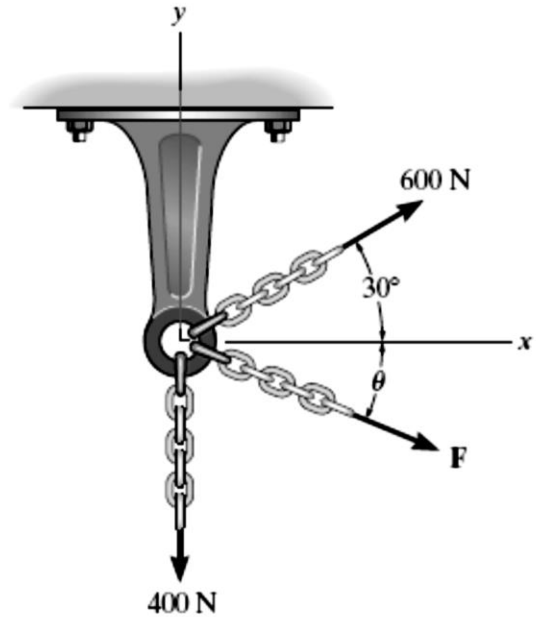
$$\frac{\sin(90^\circ - \theta)}{8} = \frac{\sin 45^\circ}{6.571}$$

$$\theta = 30.6^\circ \qquad \textbf{Ans.}$$





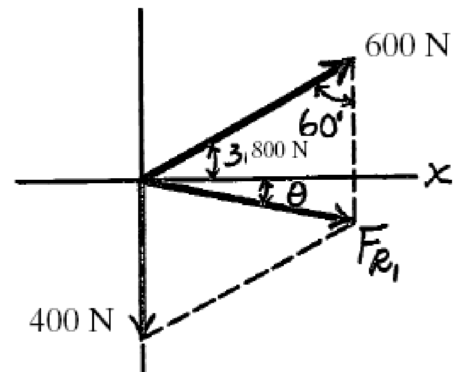
2–30. Three chains act on the bracket such that they create a resultant force having a magnitude of 1000 N. If two of the chains are subjected to known forces, as shown, determine the angle θ of the third chain measured clockwise from the positive x axis, so that the magnitude of force **F** in this chain is a *minimum*. All forces lie in the x–y plane. What is the magnitude of **F**? *Hint*: First find the resultant of the two known forces. Force **F** acts in this direction.



Cosine law:

$$F_{R1} = \sqrt{600^2 + 400^2 - 2(600)(400)\cos 60^\circ}$$

$$= 529.15 \text{ N}$$



(a)

Sine law:

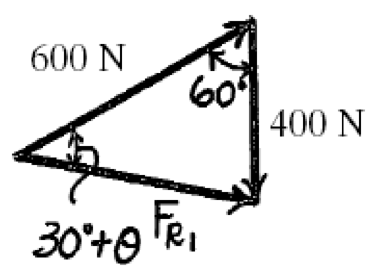
$$\frac{\sin(30^\circ + \theta)}{400} = \frac{\sin 60^\circ}{529.15} \qquad \theta = 10.9^\circ \qquad \textbf{Ans}$$

When **F** is directed along F_{R1}, F will be minimum to create the resultant force.

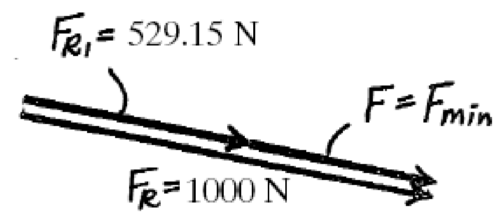
$$F_R = F_{R1} + F$$

$$1000 = 529.15 + F_{min}$$

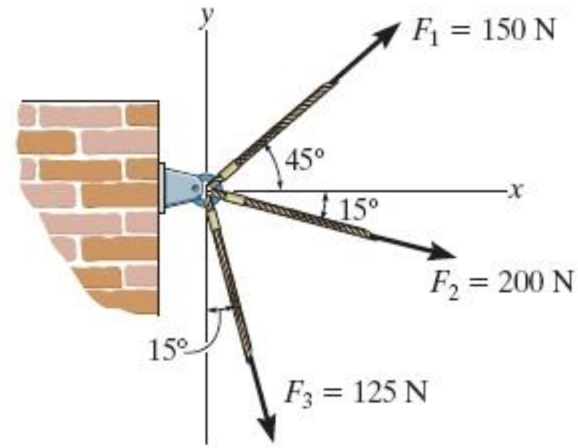
$$F_{min} = 471 \text{ N} \qquad \textbf{Ans}$$



(b)



***2–32.** Determine the magnitude of the resultant force acting on the pin and its direction measured clockwise from the positive x axis.



Rectangular components: By referring to Fig. a, the x and y components of $\mathbf{F}_1$, $\mathbf{F}_2$, and $\mathbf{F}_3$ can be written as

$(F_1)_x = 150 \cos 45° = 106.07$ N $\qquad (F_1)_y = 150 \sin 45° = 106.07$ N

$(F_2)_x = 200 \cos 15° = 193.19$ N $\qquad (F_2)_y = 200 \sin 15° = 51.76$ N

$(F_3)_x = 125 \sin 15° = 32.35$ N $\qquad (F_3)_y = 125 \cos 15° = 120.74$ N

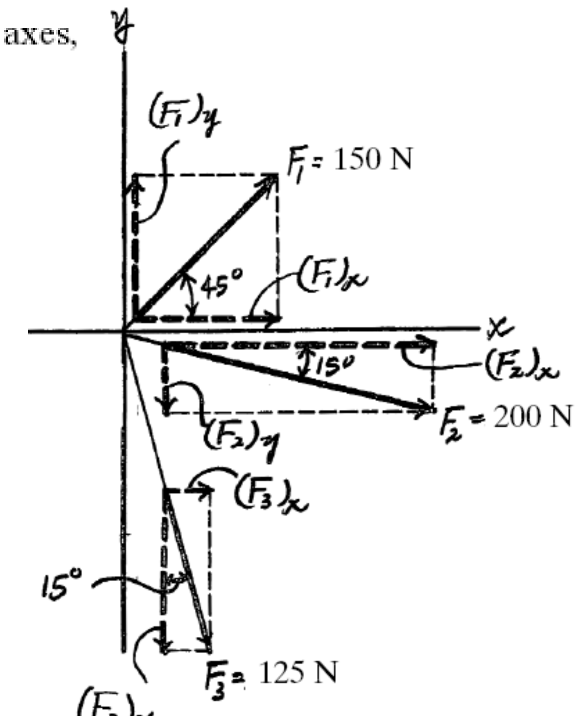
Resultant Force: Summing the force components algebraically along the x and y axes,

$$\overset{+}{\rightarrow} \Sigma (F_R)_x = \Sigma F_x; \quad (F_R)_x = 106.07 + 193.19 + 32.35 = 331.61 \text{ N} \rightarrow$$

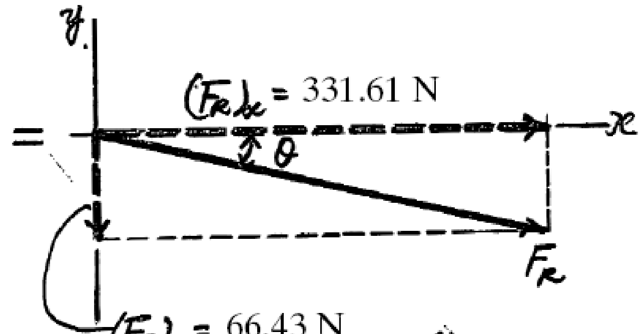
$$+\uparrow \Sigma (F_R)_y = \Sigma F_y; \quad (F_R)_y = 106.07 - 51.76 - 120.74 \qquad = -66.43 \text{ N} \downarrow$$

$$F_R = \sqrt{(F_R)_x^{\,2} + (F_R)_y^{\,2}} = \sqrt{(331.61)^2 + (-66.43)^2} = 338.2 \text{ N} \qquad \textbf{Ans}$$

$$\theta = \tan^{-1}\left[\frac{(F_R)_y}{(F_R)_x}\right] = \tan^{-1}\left(\frac{66.43}{331.61}\right) = 11.3° \qquad \textbf{Ans}$$

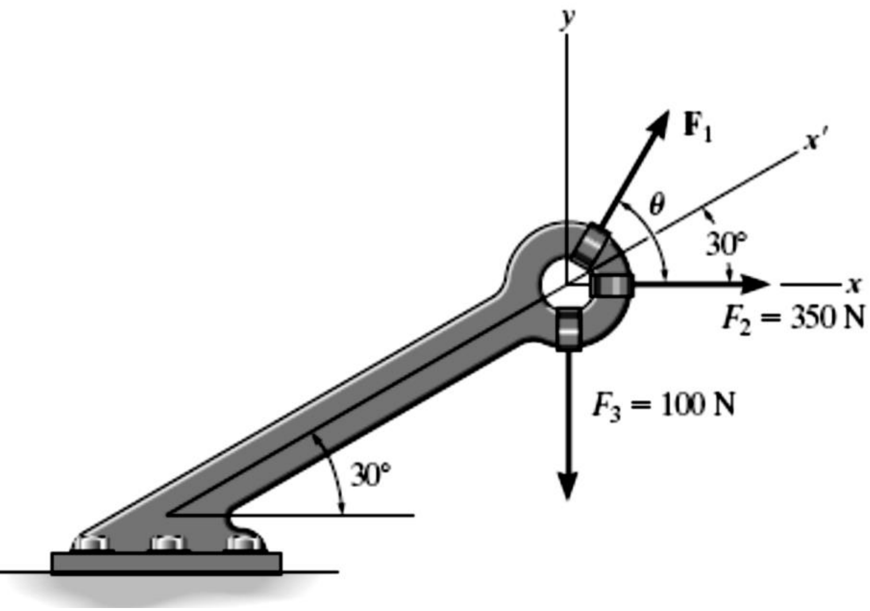


(a)



(b)

2–58. Express each of the three forces acting on the bracket in Cartesian vector form with respect to the x and y axes. Determine the magnitude and direction θ of $\mathbf{F}_1$ so that the resultant force is directed along the positive x' axis and has a magnitude of $F_R = 600$ N.



$\mathbf{F}_1 = \{F_1 \cos\theta\,\mathbf{i} + F_1 \sin\theta\,\mathbf{j}\}$ N **Ans**

$\mathbf{F}_2 = \{350\mathbf{i}\}$ N **Ans**

$\mathbf{F}_3 = \{-100\mathbf{j}\}$ N **Ans**

Require,

$$\mathbf{F}_R = 600\cos 30°\,\mathbf{i} + 600\sin 30°\,\mathbf{j}$$

$$\mathbf{F}_R = \{519.6\mathbf{i} + 300\mathbf{j}\}\text{ N}$$

$$\mathbf{F}_R = \Sigma\mathbf{F}$$

Equating the **i** and **j** components yields :

$$519.6 = F_1\cos\theta + 350$$

$$F_1\cos\theta = 169.6$$

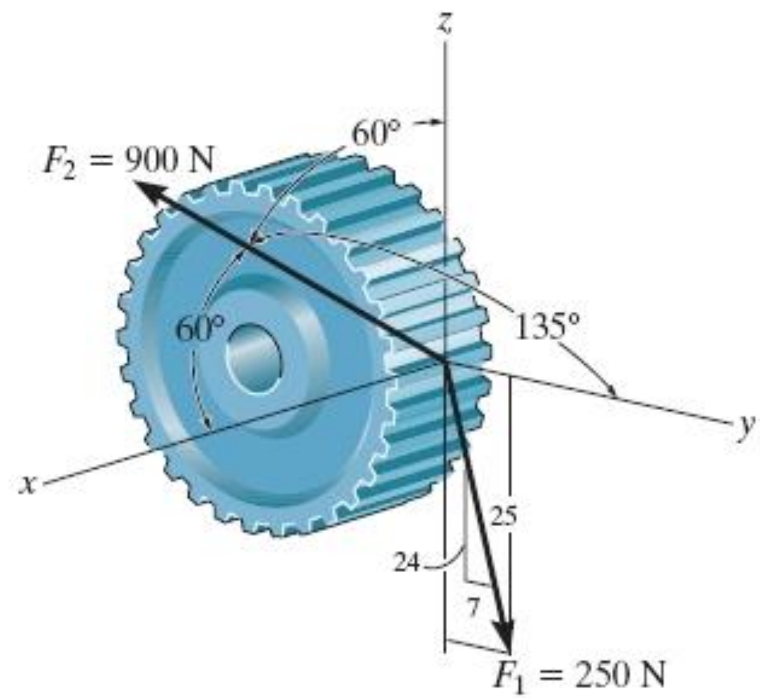
$$300 = F_1\sin\theta - 100$$

$$F_1\sin\theta = 400$$

$$\theta = \tan^{-1}\left[\frac{400}{169.6}\right] = 67.0°$$ **Ans**

$$F_1 = 434\text{ N}$$ **Ans**

***2–68.** The spur gear is subjected to the two forces caused by contact with other gears. Determine the resultant of the two forces and express the result as a Cartesian vector.



$$\mathbf{F}_1 = \frac{7}{25}(250)\mathbf{j} - \frac{24}{25}(250)\mathbf{k} = \{70\mathbf{j} - 240\mathbf{k}\}\text{ N}$$

$$\mathbf{F}_2 = 900\cos 60^\circ\mathbf{i} + 900\cos 135^\circ\mathbf{j} + 900\cos 60^\circ\mathbf{k}$$
$$= \{450\mathbf{i} - 636.4\mathbf{j} + 450\mathbf{k}\}\text{ N}$$

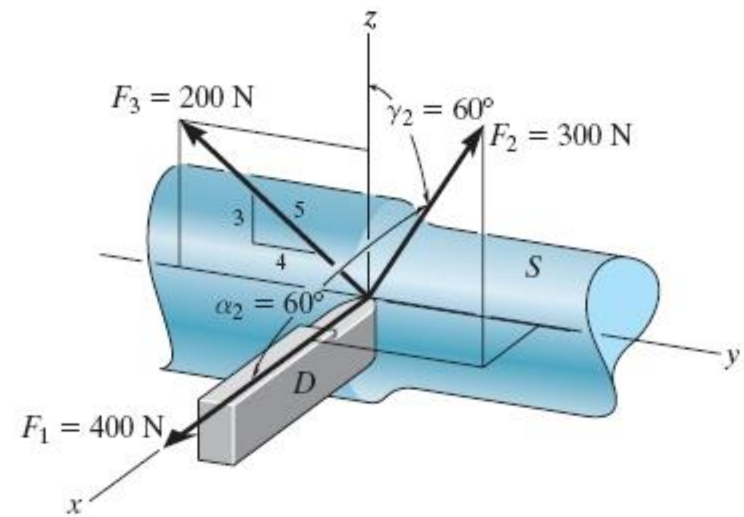
$$F_{Rx} = 900\cos 60^\circ = 450$$

$$F_{Ry} = \frac{7}{25}(250) + 900\cos 135^\circ = -566.4\text{ N}$$

$$F_{Rz} = -\frac{24}{25}(250) + 900\cos 60^\circ = 210\text{ N}$$

$$F_R = \{450\mathbf{i} - 566.4\mathbf{j} + 210\mathbf{k}\}\text{ N}$$ **Ans**

•2–73. The shaft S exerts three force components on the die D. Find the magnitude and coordinate direction angles of the resultant force. Force $\mathbf{F}_2$ acts within the octant shown.



$\mathbf{F}_1 = 400\,\mathbf{i}$

Since $\cos^2 60° + \cos^2 \beta_2 + \cos^2 60° = 1$

Solving for the positive root, $\beta_2 = 45°$

$\mathbf{F}_2 = 300\cos 60°\,\mathbf{i} + 300\cos 45°\,\mathbf{j} + 300\cos 60°\,\mathbf{k}$

$= 150\,\mathbf{i} + 212.1\,\mathbf{j} + 150\,\mathbf{k}$

$\mathbf{F}_3 = -200\left(\frac{4}{5}\right)\mathbf{j} + 200\left(\frac{3}{5}\right)\mathbf{k}$

$= -160\,\mathbf{j} + 120\,\mathbf{k}$

Then

$\mathbf{F}_R = \mathbf{F}_1 + \mathbf{F}_2 + \mathbf{F}_3 = 550\,\mathbf{i} + 52.1\,\mathbf{j} + 270\,\mathbf{k}$

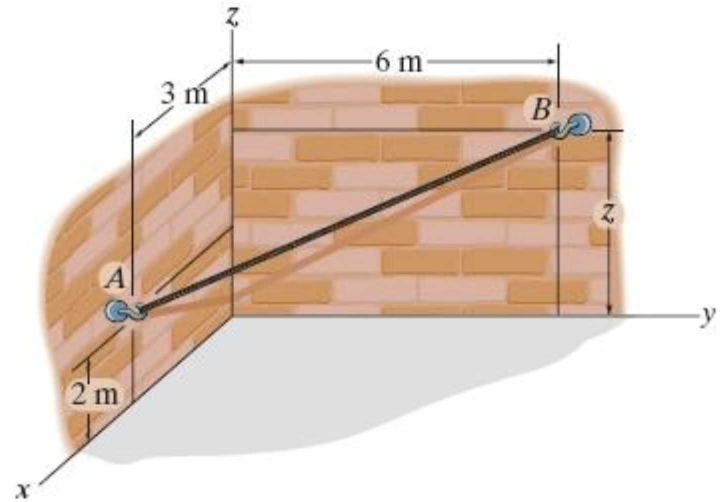
$F_R = \sqrt{(550)^2 + (52.1)^2 + (270)^2} = 614.9\text{ N} = 615\text{ N}$ **Ans**

$\alpha = \cos^{-1}\left(\frac{550}{614.9}\right) = 26.6°$ **Ans**

$\beta = \cos^{-1}\left(\frac{52.1}{614.9}\right) = 85.1°$ **Ans**

$\gamma = \cos^{-1}\left(\frac{270}{614.9}\right) = 64.0°$ **Ans**

2–86. Determine the position vector **r** directed from point A to point B and the length of cord AB. Take $z = 4$ m.



Position Vector: The coordinates for points A and B are $A(3, 0, 2)$ m and $B(0, 6, 4)$ m,

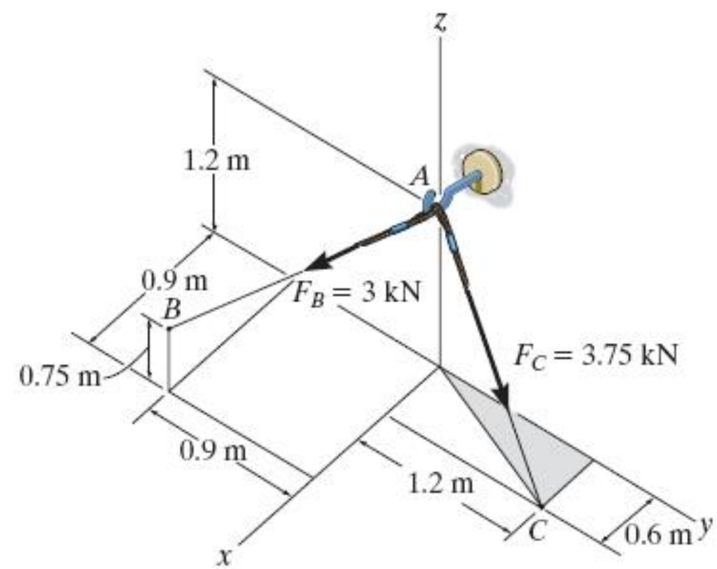
$$\mathbf{r}_{AB} = (0-3)\mathbf{i} + (6-0)\mathbf{j} + (4-2)\mathbf{k}$$

$$= \{-3\mathbf{i} + 6\mathbf{j} + 2\mathbf{k}\} \text{ m} \qquad \textbf{Ans.}$$

The length of cord AB is

$$r_{AB} = \sqrt{(-3)^2 + 6^2 + 2^2} = 7 \text{ m} \qquad \textbf{Ans.}$$

•**2–89.** Determine the magnitude and coordinate direction angles of the resultant force acting at A.



Unit Vectors: The coordinate points A, B, and C are shown in Fig. a. Thus,

$$\mathbf{u}_B = \frac{\mathbf{r}_B}{r_B} = \frac{(0.9-0)\mathbf{i} + (-0.9-0)\mathbf{j} + (0.75-1.2)\mathbf{k}}{\sqrt{(0.9-0)^2 + (-0.9-0)^2 + (0.75-1.2)^2}}$$

$$= \frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}$$

$$\mathbf{u}_C = \frac{\mathbf{r}_C}{r_C} = \frac{(0.6-0)\mathbf{i} + (1.2-0)\mathbf{j} + (0-1.2)\mathbf{k}}{\sqrt{(0.6-0)^2 + (1.2-0)^2 + (0-1.2)^2}}$$

$$= \frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}$$

Force Vectors: Multiplying the magnitude of the force with its unit vector, we have

$$\mathbf{F}_B = F_B\mathbf{u}_B = 3\left(\frac{2}{3}\mathbf{i} - \frac{2}{3}\mathbf{j} - \frac{1}{3}\mathbf{k}\right) = \{2\mathbf{i} - 2\mathbf{j} - 1\mathbf{k}\}\text{ kN}$$ **Ans.**

$$\mathbf{F}_C = F_C\mathbf{u}_C = 3.75\left(\frac{1}{3}\mathbf{i} + \frac{2}{3}\mathbf{j} - \frac{2}{3}\mathbf{k}\right) = \{1.25\mathbf{i} + 2.5\mathbf{j} - 2.5\mathbf{k}\}\text{ kN}$$ **Ans.**

$$\mathbf{F}_R = \mathbf{F}_B + \mathbf{F}_C = 2\mathbf{i} - 2\mathbf{j} - 1\mathbf{k} + 1.25\mathbf{i} + 2.5\mathbf{j} - 2.5\mathbf{k}$$

$$\mathbf{F}_R = \{3.25\mathbf{i} + 0.5\mathbf{j} - 3.5\mathbf{k}\}\text{ kN}$$

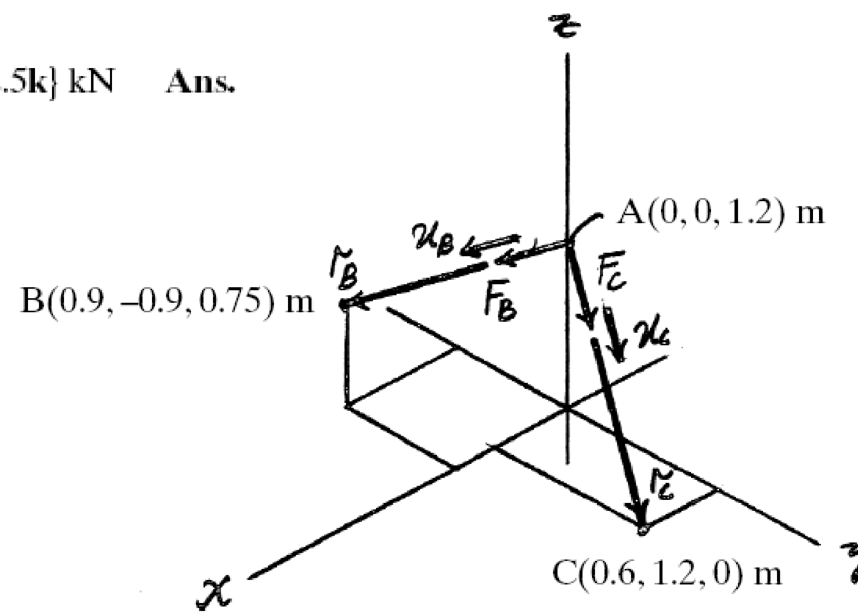
$$F_R = \sqrt{(3.25)^2 + (0.5)^2 + (-3.5)^2} = 4.80\text{ kN}$$

$$\alpha = \cos^{-1}\left[\frac{(F_R)_x}{F_R}\right] = \cos^{-1}\left(\frac{3.25}{4.80}\right) = 47.4^\circ$$

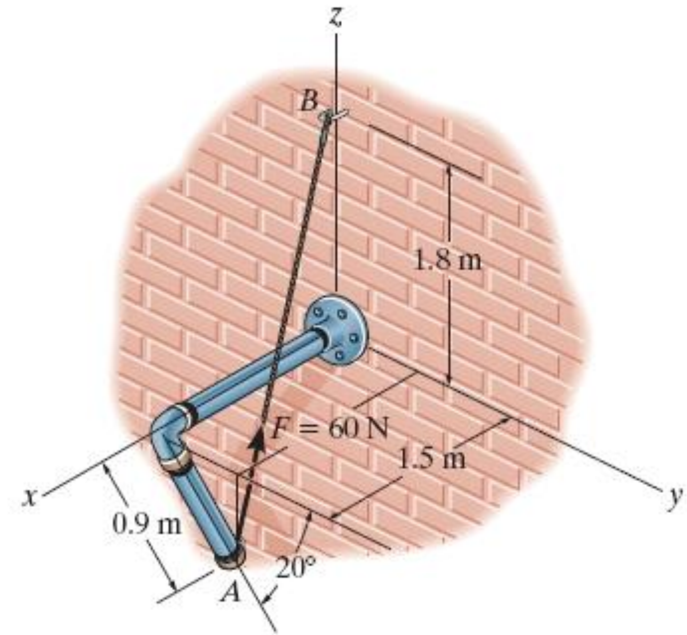
$$\beta = \cos^{-1}\left[\frac{(F_R)_y}{F_R}\right] = \cos^{-1}\left(\frac{0.5}{4.80}\right) = 84.0^\circ$$

$$\gamma = \cos^{-1}\left[\frac{(F_R)_z}{F_R}\right] = \cos^{-1}\left(\frac{-3.5}{4.80}\right) = 137^\circ$$

Ans.



2–107. The pipe is supported at its end by a cord AB. If the cord exerts a force of $F = 60$ N on the pipe at A, express this force as a Cartesian vector.



Unit Vector : The coordinates of point A are

$$A\ (1.5, 0.9\cos 20°, -0.9\sin 20°)\text{ m} = A\ (1.5, 0.8457, -0.3078)\text{ m}$$

Then

$$\mathbf{r}_{AB} = \{[0 - 1.5]\mathbf{i} + (0 - 0.8457)\mathbf{j} + (1.8\text{-}(0.3078)\mathbf{k}\}\text{ m}$$

$$= \{-1.5\mathbf{i} - 0.8457\mathbf{j} + 2.1078\mathbf{k}\}\text{ m}$$

$$r_{AB} = \sqrt{(-1.5)^2 + (-0.8457)^2 + 2.1078^2} = 2.7218\text{ m}$$

$$\mathbf{u}_{AB} = \frac{\mathbf{r}_{AB}}{r_{AB}} = \frac{-1.5\mathbf{i} - 0.8457\mathbf{j} + 2.1078\mathbf{k}}{2.7218}$$

$$= -0.5511\mathbf{i} - 0.3107\mathbf{j} + 0.7744\mathbf{k}$$

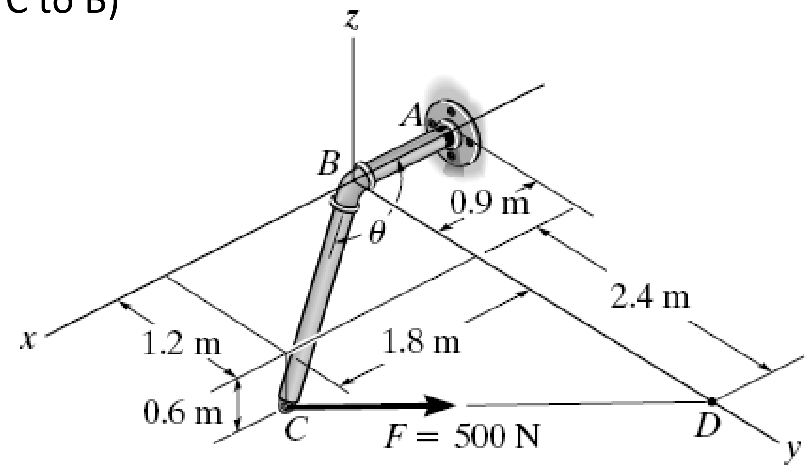
Force Vector :

$$\mathbf{F} = F\mathbf{u}_{AB} = 60\{-0.5511\mathbf{i} - 0.3107\mathbf{j} + 0.7744\mathbf{k}\}\text{ N}$$

$$= \{-33.1\mathbf{i} - 18.6\mathbf{j} + 46.5\mathbf{k}\}\text{ N}$$ **Ans**

***2–136.** Determine the magnitude of the projected component of the 500-N force acting along the axis BC of the pipe.

(from C to B)



Force Vector :

$$\mathbf{u}_{CD} = \frac{(0-1.8)\mathbf{i} + (3.6-1.2)\mathbf{j} + [0-(-0.6)]\mathbf{k}}{\sqrt{(0-1.8)^2 + (3.6-1.2)^2 + [0-(-0.6)]^2}}$$

$$= -0.5883\mathbf{i} + 0.7845\mathbf{j} + 0.1961\mathbf{k}$$

$$\mathbf{F} = F\mathbf{u}_{CD} = 500(-0.5883\mathbf{i} + 0.7845\mathbf{j} + 0.1961\mathbf{k})$$

$$= \{-294.17\mathbf{i} + 392.23\mathbf{j} + 98.058\mathbf{k}\}\ \text{N}$$

Unit Vector : The unit vector along CB is

$$\mathbf{u}_{CB} = \frac{(0-1.8)\mathbf{i} + (0-1.2)\mathbf{j} + [0-(-0.6)]\mathbf{k}}{\sqrt{(0-1.8)^2 + (0-1.2)^2 + [0-(-0.6)]^2}}$$

$$= -0.8018\mathbf{i} - 0.5345\mathbf{j} + 0.2673\mathbf{k}$$

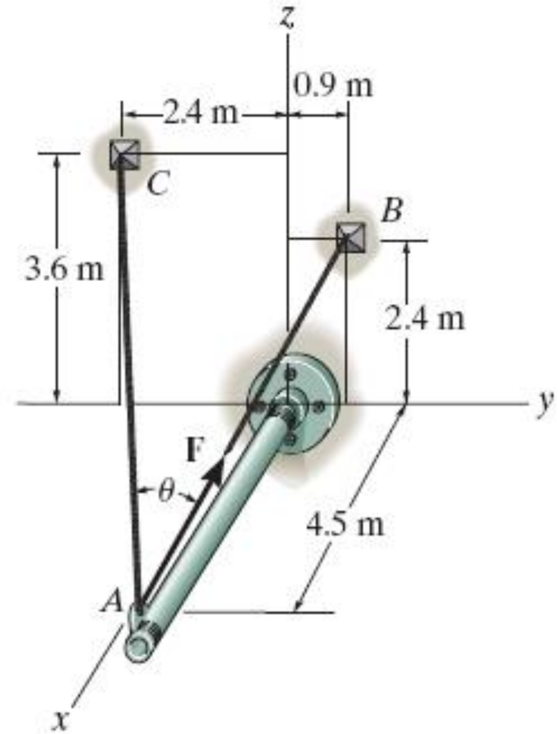
Projected Component of* F *Along CB:

$$F_{CB} = \mathbf{F}\cdot\mathbf{u}_{CB} = (-294.17\mathbf{i} + 392.23\mathbf{j} + 98.058\mathbf{k})\cdot(-0.8018\mathbf{i} - 0.5345\mathbf{j} + 0.2673\mathbf{k})$$

$$= (-294.17)(-0.8018) + (392.23)(-0.5345) + (98.058)(0.2673)$$

$$= 52.43\ \text{N} \qquad \textbf{Ans}$$

•2–129. Determine the angle θ between cables AB and AC.



Position Vector:

$$\mathbf{r}_{AB} = \{(0-4.5)\mathbf{i} + (0.9-0)\mathbf{j} + (2.4-0)\mathbf{k}\}\text{ m}$$
$$= \{-4.5\mathbf{i} + 0.9\mathbf{j} + 2.4\mathbf{k}\}\text{ m}$$

$$\mathbf{r}_{AC} = \{(0-4.5)\mathbf{i} + (-2.4-0)\mathbf{j} + (3.6-0)\mathbf{k}\}\text{ m}$$
$$= \{-4.5\mathbf{i} -2.4)\mathbf{j} + 3.6\mathbf{k}\}\text{ m}$$

The magnitudes of the position vectors are

$$\mathbf{r}_{AB} = \sqrt{(-4.5)^2 + (0.9)^2 + (2.4)^2} = 5.1788\text{ m}$$

$$\mathbf{r}_{AC} = \sqrt{(-4.5)^2 + (-2.4)^2 + (3.6)^2} = 6.2426\text{ m}$$

The Angles Between Two Vectors $\boldsymbol{\theta}$*:*

$$\mathbf{r}_{AB}\cdot\mathbf{r}_{AC} = (-4.5\mathbf{i} + 0.9\mathbf{j} + 2.4\mathbf{k})\cdot(-4.5\mathbf{i} - 2.4\mathbf{j} + 3.6\mathbf{k})$$
$$= (-4.5)(-4.5) + (0.9)(-2.4) + (2.4)(3.6)$$
$$= 26.73\text{ m}^2$$

Then,

$$\theta = \cos^{-1}\left(\frac{\mathbf{r}_{AB}\cdot\mathbf{r}_{AC}}{r_{AB}r_{AC}}\right) = \cos^{-1}\left[\frac{26.73}{5.1788\,(6.2426)}\right] = 34.2^\circ \qquad \textbf{Ans}$$