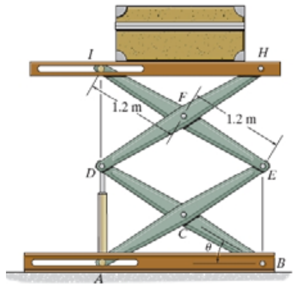
•11–1. The 200-kg crate is on the lift table at the position $\theta = 30°$. Determine the force in the hydraulic cylinder AD for equilibrium. Neglect the mass of the lift table's components.



Free Body Diagram : Fig. *a*

Virtual Displacements :

$y_D = 2.4\sin\theta \qquad \delta y_D = 2.4\cos\theta\,\delta\theta \qquad (1)$

$y_J = 2(2.4\sin\theta) + b \qquad \delta y_J = 4.8\cos\theta\,\delta\theta \qquad (2)$

Virtual – Work Equation :

$\delta U = 0; \qquad F_{AD}\,\delta y_D + [-200(9.81)\delta y_J] = 0 \qquad (3)$

Substituting Eqs. (1) and (2) into Eq. (3),

$F_{AD}(2.4\cos\theta\delta\theta) - 200(9.81)(4.8\cos\theta\delta\theta) = 0$

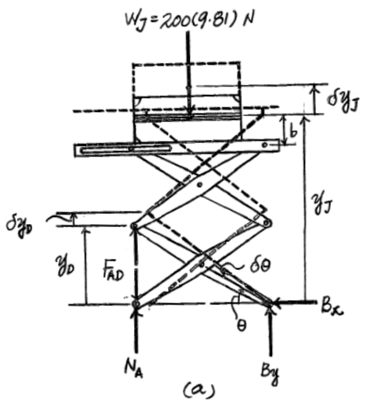
$\cos\theta\delta\theta(2.4F_{AD} - 9417.6) = 0$

Since $\cos\theta\delta\theta \neq 0$, then

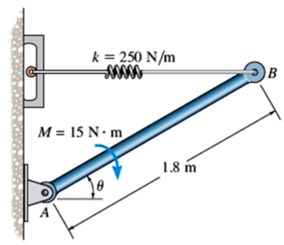
$2.4F_{AD} - 9417.6 = 0$

$F_{AD} = 3924 \text{ N} = 3.92 \text{ kN}$ **Ans.**

Note: $\mathbf{F}_{AD}$ remains constant regardless of angle θ.

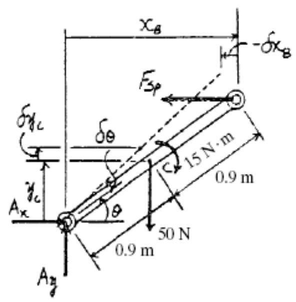


•11–5. Determine the force developed in the spring required to keep the 50 N (≈ 5-kg) uniform rod AB in equilibrium when $\theta = 35°$.



Free Body Diagram :

Note: δx_B *is positive* →



Virtual Displacements :

$x_B = 1.8\cos\theta \qquad \delta x_B = -1.8\sin\theta\,\delta\theta \qquad [1]$

$y_C = 0.9\sin\theta \qquad \delta y_C = 0.9\cos\theta\,\delta\theta \qquad [2]$

Virtual – Work Equation :

$\delta U = 0; \qquad -F_{sp}\,\delta x_B - 50\,\delta y_C - 15\,\delta\theta = 0 \qquad [3]$

Substituting Eqs. [1] and [2] into [3] yields

$(1.8F_{sp}\sin\theta - 45\cos\theta - 15)\,\delta\theta = 0 \qquad [4]$

Since $\delta\theta \neq 0$, then

$1.8F_{sp}\sin\theta - 45\cos\theta - 15 = 0$

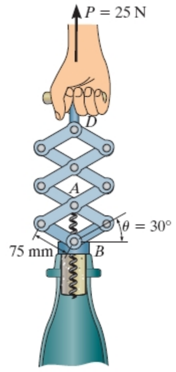
$$F_{sp} = \frac{45\cos\theta + 15}{1.8\sin\theta}$$

At the equilibrium position, $\theta = 35°$, Then

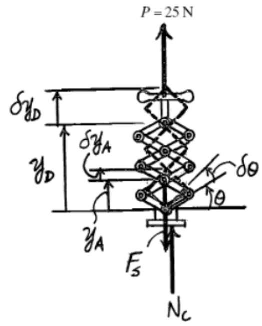
$$F_{sp} = \frac{45\cos 35° + 15}{1.8\sin 35°} = 50.2 \text{ N}$$ **Ans**

2

11–6. If a force of $P = 25$ N is applied to the handle of the mechanism, determine the force the screw exerts on the cork of the bottle. The screw is attached to the pin at A and passes through the collar that is attached to the bottle neck at B.



Free Body Diagram :



Virtual Displacements :

$y_A = 2(75 \sin \theta)$ $\qquad \delta y_A = 150 \cos \theta \, \delta\theta$ $\qquad$ (1)

$y_D = 6(75 \sin \theta)$ $\qquad \delta y_D = 450 \cos \theta \, \delta\theta$ $\qquad$ (2)

Virtual – Work Equation :

$\delta U = 0;$ $\qquad P\,\delta y_D + (-F_s\,\delta y_A) = 0$ $\qquad$ (3)

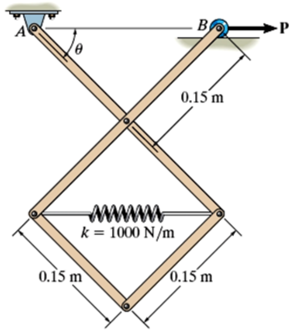
Substituting $P = 25$ N, Eqs. (1) and (2) into Eq. (3),

$$25(450 \cos \theta\,\delta\theta) - F_s(150 \cos \theta\,\delta\theta) = 0$$
$$\cos \theta\,\delta\theta(75 - F_s) = 0$$

Since $\cos \theta\,\delta\theta \neq 0$, then

$$75 - F_s = 0$$
$$F_s = 75 \text{ N}$$ **Ans.**

11–7. The pin-connected mechanism is constrained at A by a pin and at B by a roller. If $P = 50$ N, determine the angle θ for equilibrium. The spring is unstretched when $\theta = 45°$. Neglect the weight of the members.



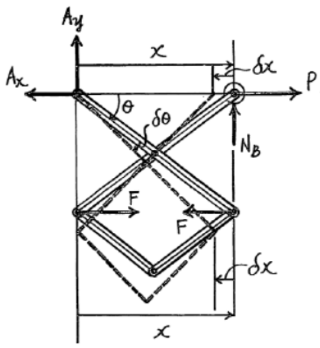
$x = 2(0.15) \cos \theta$

$F_s = ks;$ $\qquad F = 1000(0.3 \cos \theta - 0.3 \cos 45°)$

$\delta U = 0;$ $\qquad -F\,\delta x + P\,\delta x = 0$

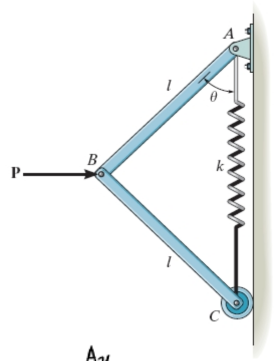
$-1000(0.3 \cos \theta - 0.3 \cos 45°) + 50 = 0$

$\theta = 29.1°$ **Ans**



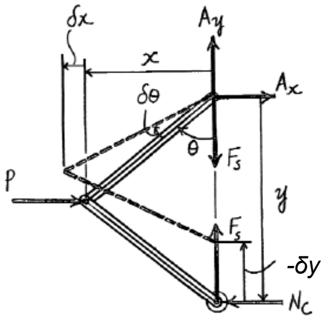
4

11–11. If the spring has a stiffness k and an unstretched length l_0, determine the force P when the mechanism is in the position shown. Neglect the weight of the members.



$$F_s = k(2l\cos\theta - l_0)$$
$$x = l\sin\theta, \quad \delta x = l\cos\theta(\delta\theta)$$
$$y = 2l\cos\theta, \quad \delta y = -2l\sin\theta(\delta\theta)$$
$$\delta U = 0; \qquad -P\delta x - F_s\delta y = 0$$
$$-Pl\cos\theta(\delta\theta) + F_s 2l\sin\theta(\delta\theta) = 0$$

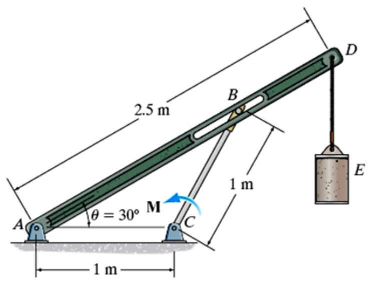
$$P = 2k\tan\theta(2l\cos\theta - l_0) \qquad \text{Ans}$$



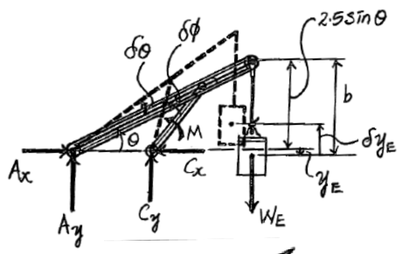
5

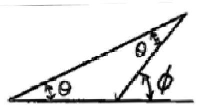
Note: *δy is positive ↓*

***11–24.** Determine the magnitude of the couple moment **M** required to support the 20-kg cylinder in the configuration shown. The smooth peg at B can slide freely within the slot. Neglect the mass of the members.



Free Body Diagram :





6

Virtual Displacements :

$y_E = b - 2.5\sin\theta \qquad \delta y_E = -2.5\cos\theta\,\delta\theta \qquad (1)$

From the geometry shown in Fig. *b*, we obtain

$\phi = 2\theta \qquad \delta\phi = 2\delta\theta \qquad (2)$

Virtual – Work Equation :

$\delta U = 0; \qquad M\delta\phi + W_E\,\delta y_E = 0 \qquad (3)$

Substituting $W_E = 20(9.81)$ N and Eqs. (1) and (2) into Eq. (3),

$$M(2\delta\theta) + 20(9.81)(-2.5\cos\theta\,\delta\theta) = 0$$
$$\delta\theta(2M - 490.5\cos\theta) = 0$$

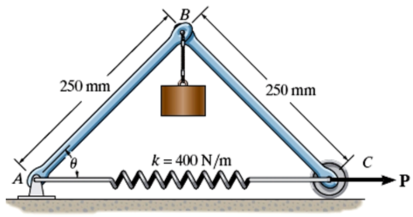
Since $\delta\theta \neq 0$, then

$$2M - 490.5\cos\theta = 0$$
$$M = 245.25\cos\theta$$

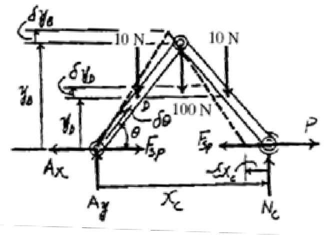
At $\theta = 30°$,

$$M = 245.25\cos 30° = 212\ \text{N}\cdot\text{m} \qquad \textbf{Ans.}$$

*11–52. The uniform links AB and BC each weigh 10 N and the cylinder weighs 100 N. Determine the horizontal force P required to hold the mechanism at $\theta = 45°$. The spring has an unstretched length of 150 mm.



Free Body Diagram :



Virtual Displacements :

$y_B = 250 \sin\theta$	$\delta y_B = 250 \cos\theta\, \delta\theta$	[1]
$y_D = 125 \sin\theta$	$\delta y_D = 125 \cos\theta\, \delta\theta$	[2]
$x_C = 2(250 \cos\theta)$	$\delta x_C = -500 \sin\theta\, \delta\theta$	[3]

Virtual – Work Equation :

$\delta U = 0;$

$$-F_{sp}\,\delta x_C - 2(10\,\delta y_D) - 100\,\delta y_B + P\,\delta x_C = 0 \qquad [4]$$

Substituting Eqs. [1], [2] and [3] into [4] yields

$$(500\,F_{sp} \sin\theta - 500\,P \sin\theta - 27500 \cos\theta)\,\delta\theta = 0 \qquad [5]$$

However, from the spring formula, $F_{sp} = kx = 0.400[2(250 \cos\theta) - 150] = 200 \cos\theta - 60$. Substituting this value into Eq. [5] yields

$$(100{,}000 \sin\theta \cos\theta - 30000 \sin\theta - 27500 \cos\theta - 500\,P \sin\theta)\,\delta\theta = 0$$

Since $\delta\theta \neq 0$, then $\quad 100{,}000 \sin\theta \cos\theta - 30000 \sin\theta - 27500 \cos\theta - 500\,P \sin\theta = 0$

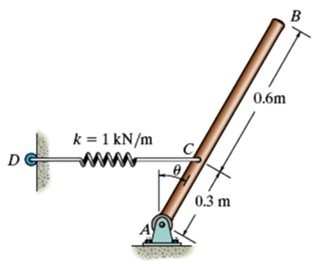
$$P = 200 \cos\theta - 55 \cot\theta - 6$$

At the equilibrium position, $\theta = 45°$. Then

$$P = 200 \cos 45° - 55 \cot 45° - 60 = 26.42 \text{ N} \qquad \textbf{Ans}$$

7

*11–56. The uniform rod AB has a weight of 50 N. If the spring DC is unstretched when $\theta = 0°$, determine the angle θ for equilibrium using the principle of virtual work. The spring always remains in the horizontal position due to the roller guide at D.



$y_w = 0.45 \cos\theta$	$\delta y_w = -0.45 \sin\theta\, \delta\theta$
$x_F = 0.3 \sin\theta$	$\delta x_F = 0.3 \cos\theta\, \delta\theta$

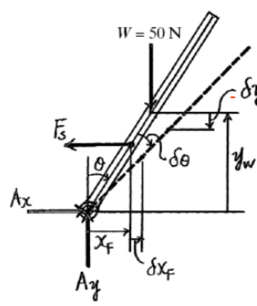
$\delta U = 0; \qquad -W\,\delta y_w - F_s\,\delta x_F = 0$

$$-50(-0.45 \sin\theta\,\delta\theta) - F_s(0.3 \cos\theta\,\delta\theta) = 0$$

$$\delta\theta(22.5 \sin\theta - 0.3\,F_s \cos\theta) = 0$$

Since $\delta\theta \neq 0$, then $\quad 22.5 \sin\theta - 0.3\,F_s \cos\theta = 0 \qquad (1)$

$F_s = kx \quad$ where $\quad x = 0.3 \sin\theta \qquad (2)$



$Fs = 1000(0.3 \sin\theta) = 300 \sin\theta$

Substituting Eq. (2) into (1) yields :

$$22.5 \sin\theta - (90 \sin\theta) \cos\theta = 0$$

$$\sin\theta(22.5 - 90 \cos\theta) = 0$$

$\sin\theta = 0 \qquad \theta = 0° \qquad$ **Ans**

$22.5 - 90 \cos\theta = 0 \qquad \theta =$ **75.5°** $\qquad$ **Ans**

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