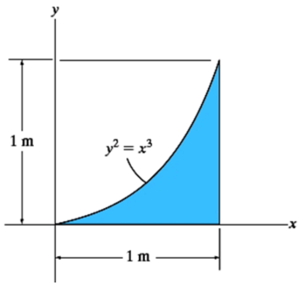
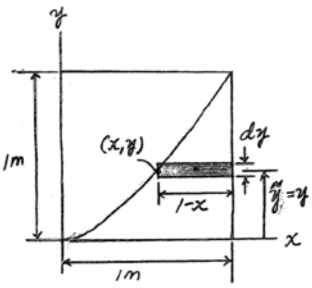
10–3. Determine the moment of inertia of the area about the x axis.



The area of the rectangular differential element in Fig. *a* is $dA = (1-x)\,dy$. Since $x = y^{2/3}$, then $dA = (1 - y^{2/3})\,dy$.



(a)

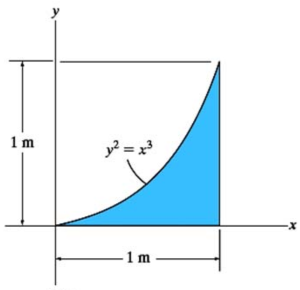
$$I_x = \int_A y^2 dA$$

$$= \int_0^{1\,\text{m}} y^2\left[1 - y^{2/3}\right] dy$$

$$= \int_0^{1\,\text{m}} (y^2 - y^{8/3})\, dy$$

$$= \left(\frac{y^3}{3} - \frac{3}{11} y^{11/3}\right)\Bigg|_0^{1\,\text{m}} = 0.0606\ \text{m}^4$$ 1

***10–4.** Determine the moment of inertia of the area about the y axis.



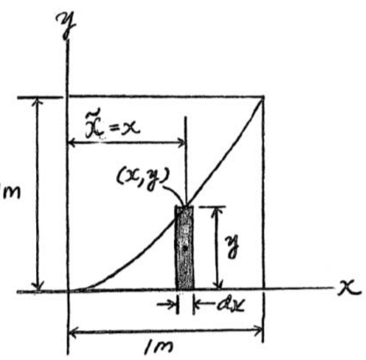
The area of the rectangular differential element in Fig. *a* is $dA = y\,dx = x^{3/2}\,dx$.

$$I_y = \int_A x^2 dA$$

$$= \int_0^{1\,\text{m}} x^2\left(x^{3/2}\right) dx$$

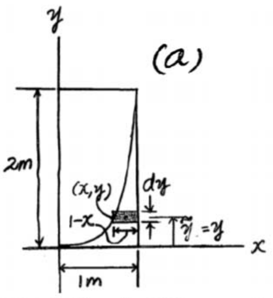
$$= \int_0^{1\,\text{m}} x^{7/2}\, dx$$

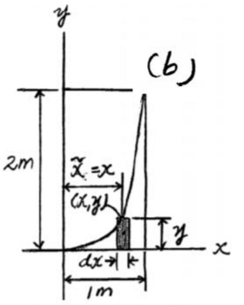
$$= \left(\frac{2}{9} x^{9/2}\right)\Bigg|_0^{1\,\text{m}} = 0.222\ \text{m}^4$$

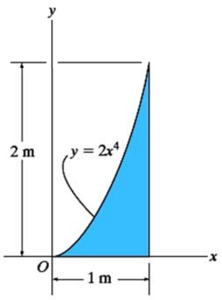


2

•10–9. Determine the polar moment of inertia of the area about the z axis passing through point O.







The moment of inertia of the area about the x and y axes will be determined using the rectangular differential element in Figs. a and b. The area of these two elements are

$$dA = (1-x)\,dy = \left[1-\left(\frac{y}{2}\right)^{1/4}\right]dy \text{ and } dA = y\,dx = 2x^4\,dx.$$

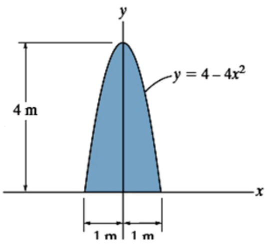
$$I_x = \int_A y^2 dA = \int_0^{2\,\text{m}} y^2\left[1-\left(\frac{y}{2}\right)^{1/4}\right]dy = \int_0^{2\,\text{m}}\left[y^2-\left(\frac{1}{2}\right)^{1/4}y^{9/4}\right]dy = 0.2051\ \text{m}^4$$

$$I_y = \int_A x^2 dA = \int_0^{1\,\text{m}} x^2\left(2x^4 dx\right) = \int_0^{1\,\text{m}} 2x^6\,dx = \left(\frac{2}{7}x^7\right)\Big|_0^{1\,\text{m}} = 0.2857\ \text{m}^4$$

the polar moment of inertia of the area about the z axis is

$$J_O = I_x + I_y = 0.2051 + 0.2857 = 0.491\ \text{m}^4$$ 3

10–14. Determine the moment of inertia of the area about the x axis. Solve the problem in two ways, using rectangular differential elements: (a) having a thickness of dx, and (b) having a thickness of dy.



a) ***Differential Element :*** The area of the differential element parallel to y axis is $dA = ydx$. The moment of inertia of this element about x axis is

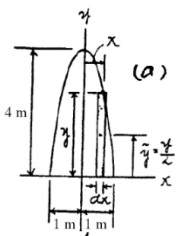
$$dI_x = d\bar{I}_{x'} + dA\,\bar{y}^2 = \frac{1}{12}(dx)y^3 + ydx\left(\frac{y}{2}\right)^2 = \frac{1}{3}(4-4x^2)^3\,dx$$

$$= \frac{1}{3}(-64x^6 + 192x^4 - 192x^2 + 64)\,dx$$

Moment of Inertia : Performing the integration, we have

$$I_x = \int dI_x = \frac{1}{3}\int_{-1\,\text{m}}^{1\,\text{m}} (-64x^6 + 192x^4 - 192x^2 + 64)\,dx$$

$$= \frac{1}{3}\left(-\frac{64}{7}x^7 + \frac{192}{5}x^5 - \frac{192}{3}x^3 + 64x\right)\Big|_{-1\,\text{m}}^{1\,\text{m}} = 19.5\ \text{m}^4 \qquad \textbf{Ans}$$

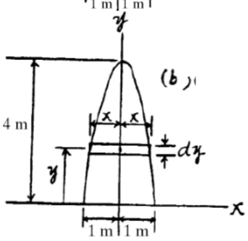


b) ***Differential Element :*** Here, $x = \frac{1}{2}\sqrt{4-y}$. The area of the differential element parallel to x axis is $dA = 2xdy = \sqrt{4-y}\,dy$.

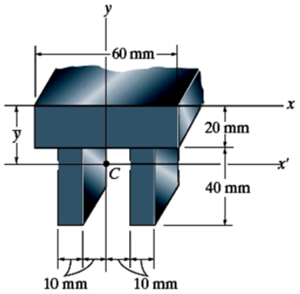
Moment of Inertia : Performing the integration, we have

$$I_x = \int_A y^2\,dA = \int_0^{4\,\text{m}} y^2\sqrt{4-y}\,dy$$

$$= \left[-\frac{2y^2}{3}(4-y)^{\frac{3}{2}} - \frac{8y}{15}(4-y)^{\frac{5}{2}} - \frac{16}{105}(4-y)^{\frac{7}{2}}\right]\Big|_0^{4\,\text{m}} = 19.5\ \text{m}^4 \qquad \textbf{Ans}$$



10–27. Determine the distance $\bar{y}$ to the centroid of the beam's cross-sectional area; then find the moment of inertia about the x' axis.



Centroid :

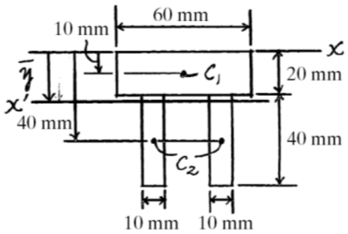
$$\bar{y} = \frac{\Sigma \tilde{y} A}{\Sigma A} = \frac{10(60)(20) + 2[40(40)(10)]}{60(20) + 2[40(10)]} = 22.0 \text{ mm} \quad \textbf{Ans}$$

Moment inertia :

$$I_{x'} = \frac{1}{12}(60)(20)^3 + 60(20)(22.0 - 10)^2$$

$$+ 2\left[\frac{1}{12}(10)(40)^3 + 10(40)(40 - 22.0)^2\right]$$

$$= 57.9\,(10^4)\text{ mm}^4 \quad \textbf{Ans}$$



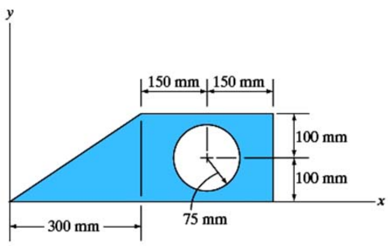
*__10–28.__ Determine the moment of inertia of the beam's cross-sectional area about the x axis.

$$I_x = \left[\frac{1}{12}(60)(20)^3 + (60)(20)(10)^2\right]$$

$$+ 2\left[\frac{1}{12}(10)(40)^3 + (40)(10)(40)^2\right]$$

$$= 155\,(10^4)\text{ mm}^4 \quad \textbf{Ans}$$

*__10–32.__ Determine the moment of inertia of the composite area about the x axis.



Moment of Inertia:

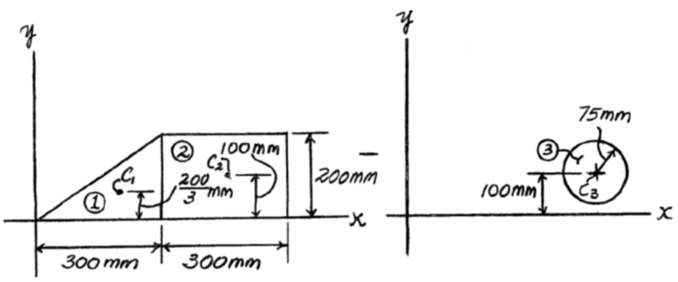
using the parallel - axis theorem.

$$I_x = \bar{I}_{x'} + A(d_y)^2$$

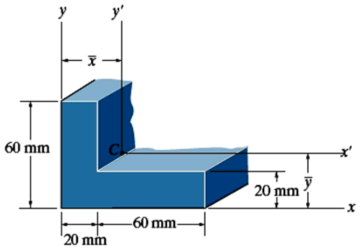
$$= \left[\frac{1}{36}(300)(200^3) + \frac{1}{2}(300)(200)\left(\frac{200}{3}\right)^2\right] + \left[\frac{1}{12}(300)(200^3) + 300(200)(100)^2\right] + \left[-\frac{\pi}{4}(75^4) + \left(-\pi(75^2)\right)(100)^2\right]$$

$$= 798(10^6)\text{ mm}^4$$

Ans.



10–43. Locate the centroid $\bar{y}$ of the cross-sectional area for the angle. Then find the moment of inertia $I_{x'}$ about the x' centroidal axis.

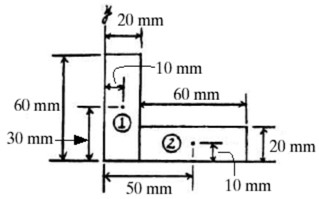


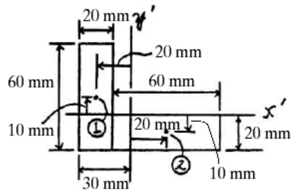
Centroid : The area of each segment and its respective centroid are tabulated below.

Segment	A (mm^2)	$\bar{y}$ (mm)	$\bar{y}A$ (mm^3)
1	60(20)	30	36000
2	60(20)	10	12000
Σ	2400		48000

Thus,

$$\bar{y} = \frac{\Sigma \bar{y}A}{\Sigma A} = \frac{48000}{2400} = 20 \text{ mm} \qquad \textbf{Ans}$$





Moment of Inertia :

Segment	A_i (mm^2)	$(d_y)_i$ (mm)	$(\bar{I}_{x'})_i$ (mm^4)	$(Ad_y^2)_i$ (mm^4)	$(I_x)_i$ (mm^4)
1	20(60)	10	$\frac{1}{12}(20)(60^3)$	12.0 (10^4)	48.0 (10^4)
2	60(20)	10	$\frac{1}{12}(60)(20^3)$	12.0 (10^4)	16.0 (10^4)

Thus,

$$I_{x'} = \Sigma (I_{x'})_i = 64.0\,(10^4)\text{ mm}^4 \qquad \textbf{Ans}$$